Foreword

The *Physics 12 Student Laboratory Manual* was developed for the Ministry of Education under the guidance of the Physics 11/12 Curriculum Revision Committee. The primary purpose of the manual is to provide students of Physics 12 with interesting and challenging investigations which will assist them in understanding the discipline of physics.

The Ministry of Education gratefully acknowledges the contributions of the Physics 11/12 Curriculum Revision Committee and the work of the writer, Ron Somers, in particular.

As the result of many useful suggestions from the field and the advent of provincial examinations, the original Student Laboratory Manual (1982) has been updated and reorganized. The reorganization project was co-ordinated by Richard Warrington of the Curriculum Development Branch of the Ministry of Education.

This Second Edition of the Physics 12 Student Laboratory Manual (1985) has been prepared to complement closely the Physics 12 Curriculum Guide (Updated version 1984), which was published in response to requests from teachers for a clarification of the intent of the Physics 11/12 Curriculum.

The Curriculum Development Branch wishes to thank the many teachers who offered suggestions, advice and technical reviews of the manuscripts.

Preface (to the 1982 Edition)

I am deeply indebted to the following members of the Physics Revision Committee, 1979/1981:

Monty Clements
Dave Gabel
Karam Gopaulsingh
Lou de Macedo·
Mike McKee
Michael Verge

Their considerable aid in developing the investigations and their constructive criticisms of the numerous drafts of the manual were invaluable to me.

My grateful appreciation is also extended to all of those teachers of physics whose good ideas I have "borrowed" and incorporated into this manual.

Ron Somers

Contents

Introduction

Performing laboratory investigations or experiments is an important part of the study of physics. Generally, physicists use controlled experiments to verify their hypotheses about the phenomena of physics. Yet it often requires more thought for them to devise an appropriate experimental check on a budding theory than to formulate the theory itself. Physicists may, for instance, have to perform their experiments on the top of a mountain, at the bottom of the ocean, or on a satellite voyage to Saturn, since some of the distinctive phenomena of physics can be observed only in a certain environment. It is worth remembering, therefore, that the study of physics need not be limited to a school classroom.

Usually laboratory investigations or experiments lead to new hypotheses requiring even more experiments. Rarely in physics does asking a single question lead to one experiment that will give a unique answer; experiments usually raise far more questions than they answer. Even experiments that completely fail to prove the expected outcome are important in that reasons must be found to explain why they "failed." The explanations sometimes lead to entirely new concepts and theories which radically alter our understanding of everyday phenomena. This is a major reason why people stay so interested in the development of physics; they never know what is going to happen next.

As you go through the investigations in this guide you will learn many of the experimental techniques used by researchers to obtain data, to control the various factors that can influence results, to analyse the data obtained, and to interpret results so that conclusions and further hypotheses can be made. You will also encounter some of the problems that confront all experimental researchers: experimental design, limitations of equipment, imprecise data, and inconclusive results. It may be in dealing with these problems that you will learn the most about physics.

The Organization of the Investigations

The investigations in this guide have each been chosen to represent one of the major topics encountered in the Physics 12 course. Each investigation is presented in several sections. The first section is an Introduction, giving you a brief background of the concepts being studied in the investigation and perhaps some of the theory involved. The Apparatus that follows is a list of the major pieces of equipment needed to perform the experimental part of the investigation. Since not all school laboratories stock exactly the same equipment you will sometimes have to decide on your own how to set up the apparatus. The next section, Procedure, briefly outlines what should be done to obtain the necessary data. The instructions in this section are often fairly general because of the differences in the equipment and the method used. The exact procedure you follow, which may vary between lab groups in the same class, should be carefully recorded as it may be very valuable in the interpretation of your results. Once you have performed the experiment, the Data Analysis will guide you in using the data obtained to find relationships among the variables studied during the experiment. These relationships are usually derived graphically and presented in the form of a mathematical relation. The Discussion of Results will then indicate the significance of your results and should guide you to a more general understanding of the ideas involved in the experiment. In addition, some indication is given of the uncertainties, the approximations, and the errors that could have affected your results. Ideas for other related investigations are given in the Additional Activities and each investigation concludes with a series of Practice Problems that allow you to apply what you have learned to the solution of specific related problems. The answers to these problems are included in Appendix 14 at the back of this manual.

Vector Kinematics

Investigation 1:

Projectile Motion

Most types of motion that occur in nature are far more complex than the straight line motion you have studied so far. The motion of a projectile can best be studied as the sum of two components of motion—one vertical and one horizontal. First, however, you must make a record of the motion.

Method 1

This method involves making and analysing a strobe photograph of a thrown ball.

Apparatus

strobe light
Polaroid camera
ball
metre stick

Procedure

1. Make a strobe photo of a ball projected horizontally off the edge of a table or thrown upwards at an angle. (See Appendix 11.)

Method 2

This method involves using Projectile Motion Analysis apparatus to create a graph of a projected ball.

Apparatus

Projectile Motion Analysis apparatus
large sheet of graph paper
carbon paper

Procedure

Follow the instructions given with the apparatus. A record of the path of a projected ball is generated by the procedure.

Method 3

This method involves using an air table to create a strobe photograph of a moving disc for analysis.

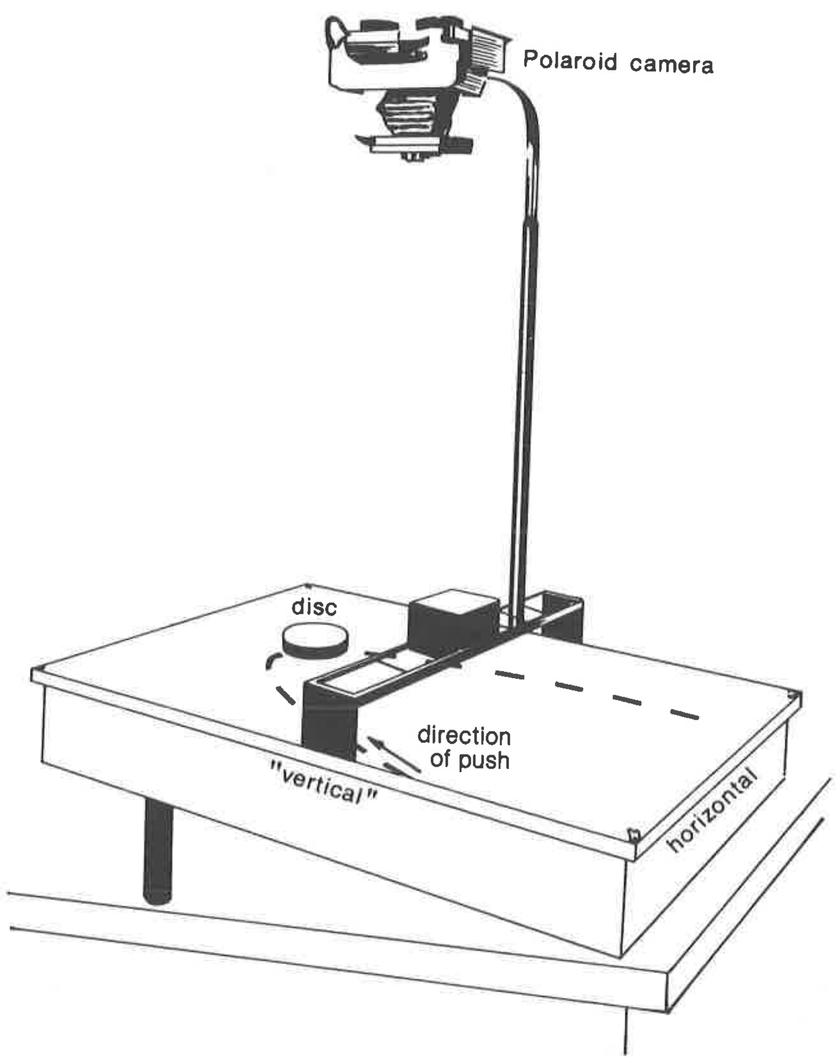
Apparatus

air table and disc
strobe light
Polaroid camera

Procedure

1. Tilt an air table slightly so that when the air puck is pushed as shown it will travel up the air table and then down again. (See Figure #001.)

#001 Air table and disc



2. Take a strobe photo of the motion. (See Appendix 11.)

Method 4

In this method, an illustration of a strobe photograph has been provided for analysis.

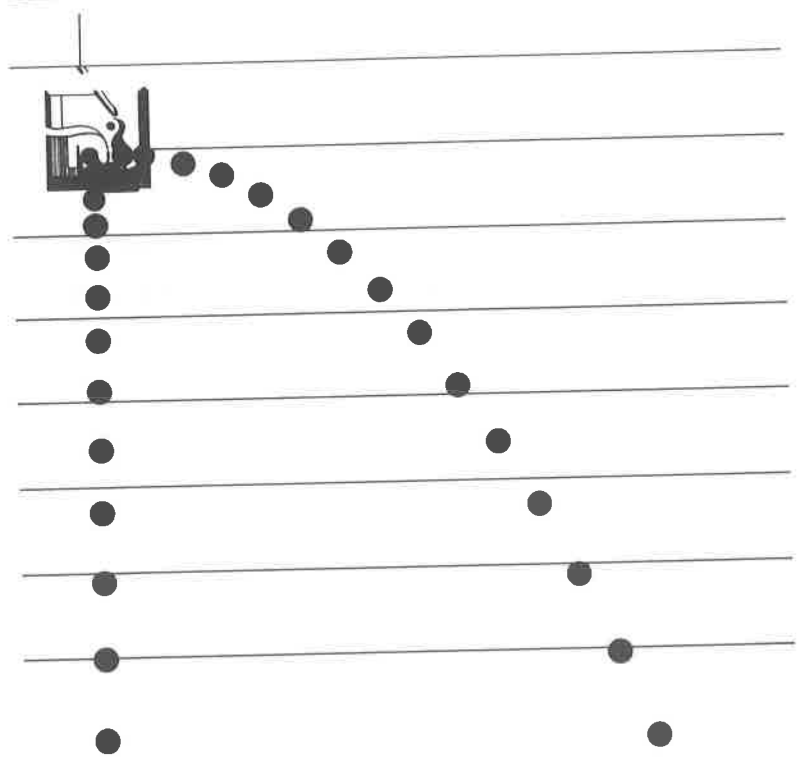
Apparatus

illustration of a strobe photo (#002)

Procedure

Illustration #002 shows two balls that have been released simultaneously, one projected horizontally and the other dropped.

#002 Illustration of a strobe photo; the lines are 14.35 cm apart and the strobe frequency is 30 Hz.

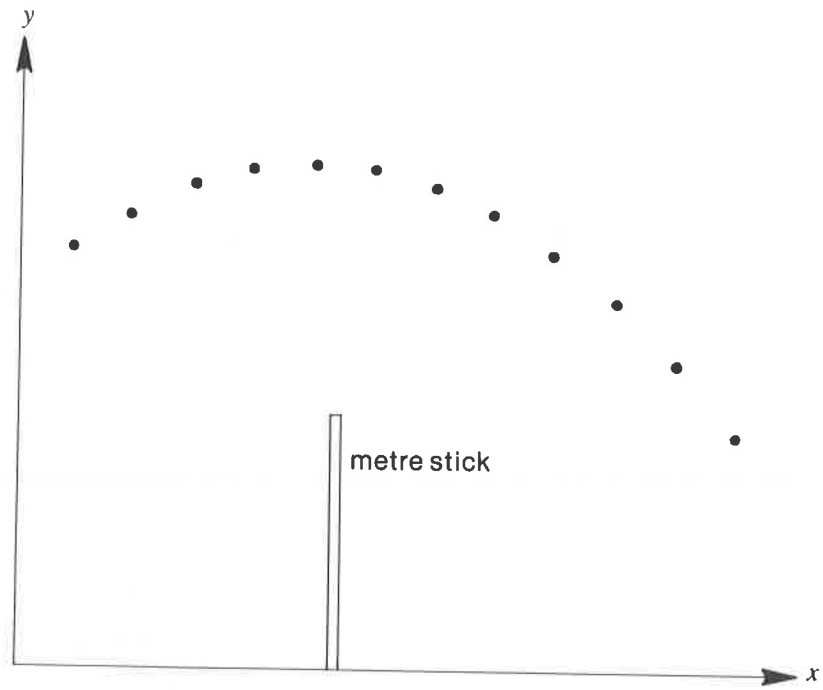


All Methods

Regardless of the apparatus and procedure used, complete the investigation as follows.

The following diagram (#003) represents a strobe illustration of a ball thrown upwards at an angle to the horizontal.

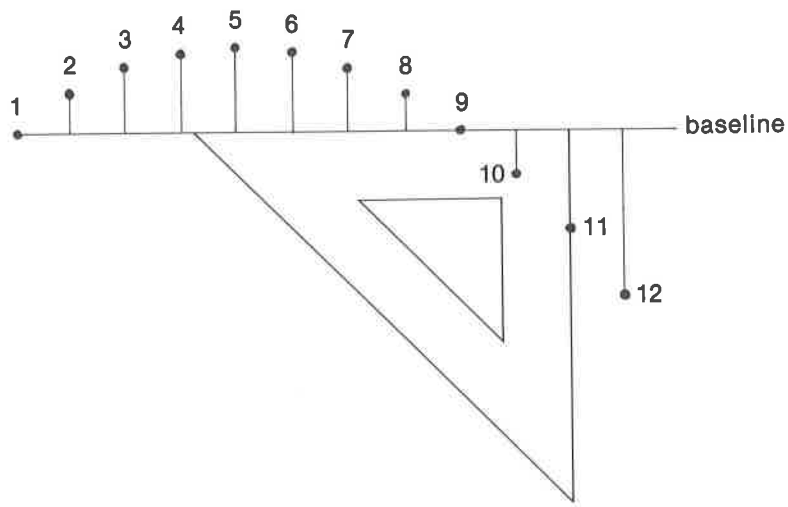
#003 Strobe illustration of the path of a projected ball.
Strobe frequency = 15 Hz.



This motion can be analysed using the following procedure:

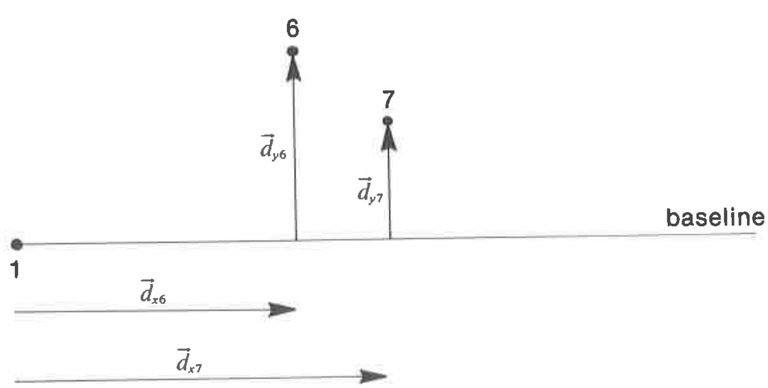
1. number the successive positions of the ball.
2. draw a line through the centre of the leftmost ball's position, parallel to the horizontal (this will be called the baseline).
3. Use a plastic right-angled triangle to draw lines from the baseline to the centre of each position of the ball as shown in figure #004.

#004 Analysing the motion of a ball: Step 3



4. For each flash, measure the horizontal and the vertical components ($\vec{d}_x$ and $\vec{d}_y$) of the ball's displacement from its original position. (Use + for upward displacements and − for downward.)

#005 Analysing the motion of a ball: Step 4



$\Delta\vec{d}_y = \vec{d}_{y7} - \vec{d}_{y6}$ and $\Delta\vec{d}_x = \vec{d}_{x7} - \vec{d}_{x6}$

5. The average vertical velocity of the ball between flashes can then be calculated by dividing $\Delta\vec{d}_y$ by $\frac{1}{15}$ s, the time between the flashes. To find the actual velocity of the ball you must first calculate the scale factor. For example, according to diagram #003, 3.2 cm of the photo is equivalent to 1.00 m. In that case, the scale factor is

$$\frac{100 \text{ cm}}{3.37 \text{ cm}} = 29.7$$

Then you can calculate the actual average vertical velocity between flash 6 and flash 7, for example, as follows.

$$v = \frac{\Delta\vec{d}_y \times 29.7}{\frac{1}{15}\text{ s}} = \frac{-0.25 \text{ cm} \times 29.7}{\frac{1}{15}\text{ s}} = -110 \text{ cm/s}$$

Although this value is the *average* vertical velocity between flash 6 and flash 7, it can be expected to be very close, if not equal, to the *instantaneous* vertical velocity midway in time between flash 6 and flash 7.

You can calculate the instantaneous horizontal velocity midway in time between two flashes in a similar way from $\Delta\vec{d}_x$.

6. Make a table with the following headings (see table #006).

t time measured in flashes of the strobe

$\vec{d}_y$ vertical component of the displacement of the ball from its original position

$\vec{v}_y$ vertical component of the ball's velocity midway between flashes

$\vec{d}_x$ horizontal component of the displacement of the ball from its original position

$\vec{v}_x$ horizontal component of the ball's velocity midway between flashes

List $\vec{d}_x$ and $\vec{d}_y$ for successive images of the ball. Calculate $\vec{v}_x$ and $\vec{v}_y$ from $\Delta\vec{d}_x$ and $\Delta\vec{d}_y$ and add them in.

7. Graph (a) $\vec{d}_x$ vs. t, (b) $\vec{d}_y$ vs. t, and (c) $\vec{v}_y$ vs. t. (See Appendix 3.)

8. Find the mathematical equation for each of your graphs. (See Appendix 4.)

#006 Table of variables

t	$\vec{d}_x$	$\vec{d}_y$	$\vec{v}_x$	$\vec{v}_y$
0				
1				
2				
3				
4				

Discussion of Results

1. Is the horizontal velocity of a projected ball constant? Use your graphs to support your explanation. What is the horizontal acceleration of a projectile?

2. If you used Methods 1, 2, or 4, calculate the vertical acceleration of the projected ball (in cm/flash^2). Convert this acceleration to the "real" acceleration of the actual ball in the laboratory. Compare your result to the accepted value of g. (See Appendix 5.)

3. Use the horizontal and vertical components of velocity to calculate the vector velocity of the projected ball or disc midway in time between any two consecutive flashes.

4. If you used Method 4, compare your graphs of vertical velocity versus time for the two balls. Do the balls fall with the same acceleration?

5. You know the general equations for uniformly accelerated motion:

   $$v = v_i + at$$
   $$d = v_it + \tfrac{1}{2}\,at^2$$

   where v and d are the instantaneous velocity and displacement of the object from its original position at time t, v_i is the initial velocity of the object, and a is its acceleration. Applying these equations to the x and y components of the motion of a projectile, we expect

   $$v_y = v_{yi} - gt$$
   $$d_x = v_{xi}t \qquad\qquad d_y = v_{yi}t - \tfrac{1}{2}\,gt^2$$

   How do the equations of your graphs compare with these equations? (See Appendix 6.)

6. How would you describe the variation with time of a projectile's
   (a) horizontal velocity?
   (b) vertical velocity?
   (c) vertical acceleration?
   (See Appendix 7.)

Additional Activities

1. Launch a projectile at 45° using various launching speeds and measure the range and the maximum height reached. Compare the values of these two quantities from one launch to another.

2. Use a pinball launcher (or a controllable slingshot) to release a ball at various angles, maintaining a constant initial speed. For each launch, measure the range of the projected ball and record the angle of projection. Try to determine the relationship involved. Also measure the maximum height reached and try to determine the relationship between this height and the angle of projection above the horizontal.

Practice Problems

Note: Throughout this book, assume that all zeroes not clearly significant are, in fact, significant.

1. A ball is dropped from a height of 80 m. How long does it take to fall?

2. A ball is thrown horizontally at a speed of 40 m/s from a cliff 80 m high.

   (a) How long does it take to strike the ground?

   (b) How far horizontally does the ball travel before striking the ground?

3. A golfball is driven at a speed of 80 m/s at an angle of 40° from the ground. Disregard any effects due to air resistance.

   (a) What is the vertical component of the initial velocity?

   (b) What is the horizontal component of the initial velocity?

   (c) How long will the ball be in the air (if you assume that the ball strikes the ground at the same level as it leaves the ground)?

   (d) What maximum altitude will the golfball reach?

   (e) How far will the golfball go horizontally before striking the ground?

4. A stone is fired from a slingshot at an angle of 65° from the horizontal. The stone strikes the ground 8.0 s later at an altitude 30 m lower than the height at which it was released.

   (a) At what initial velocity was the stone released?

   (b) How far horizontally does the stone go before striking the ground?

5. Two archers are trying to hit a distant target. One shoots at a low angle, thinking that such a shot will be more accurate. The other shoots at a greater angle, expecting to attain greater distance. If you assume that they both shoot at the same initial speed, show that they could both hit the target. Also show that the two angles add up to 90°.

In questions of science the authority of a thousand is not worth the humble reasoning of a single individual.
Galileo Galilei

Mechanical Energy and Vector Momentum

Investigation 2:

Oblique Collisions in Two Dimensions

In any collision between objects in an isolated system, the total momentum of the objects after the collision is the same as before the collision. For two objects of masses m_1 and m_2 with velocity u_1 and u_2 respectively before a collision, this fact can be described by the following equation:

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the objects after the collision.

In practice, most systems are not isolated; there is an unbalanced force on the objects in the system from outside the system. However, the forces involved in collisions are usually *impulsive;* that is, much larger than the forces on the objects from outside the system. In that case, the momentum immediately after the collision is equal to the momentum immediately before the collision.

However, if we want to measure the momentum of an object before or after a collision, it must maintain its momentum long enough for it to be measured. In that case, we must study almost frictionless motion – the motion of pucks on an air table or pendulum bobs falling through the air.

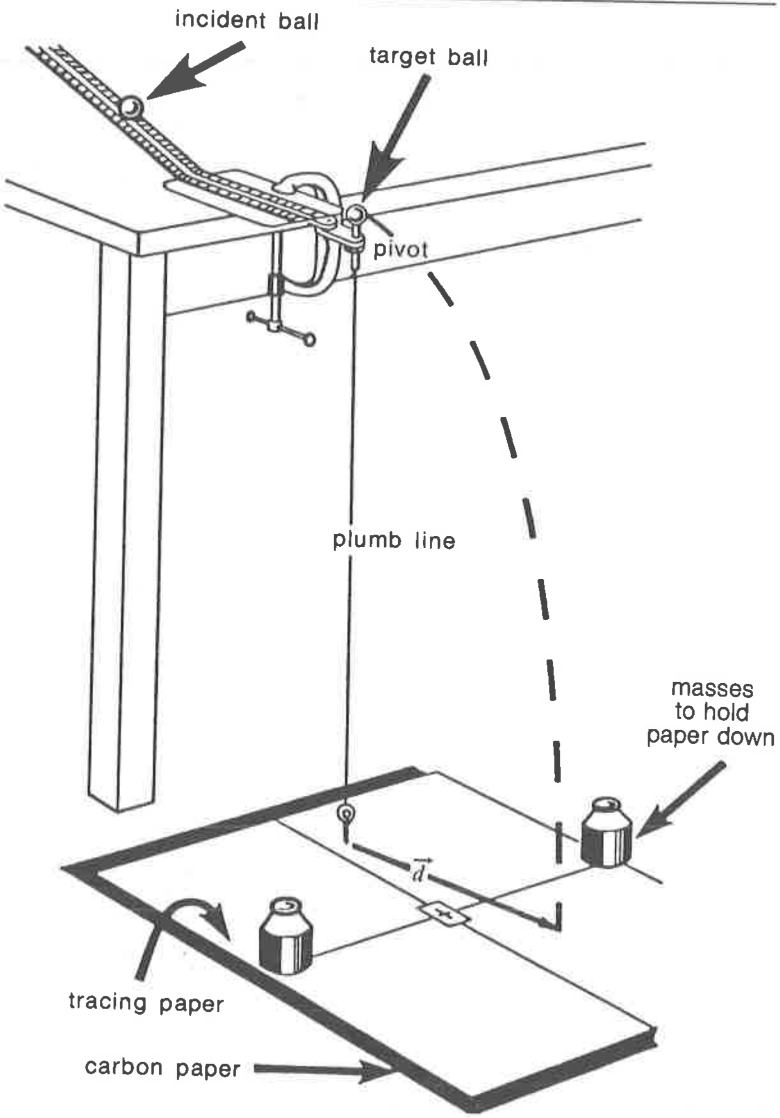
In any interaction between objects, the total energy also remains constant. In most collisions, however, some of the kinetic energy of the objects is converted to other forms of energy, particularly heat energy. Only if the colliding objects are very hard and very elastic will most of the kinetic energy be conserved. If no kinetic energy is transformed, the collision is called perfectly elastic. Other collisions can be partially elastic, in which case a percentage of the kinetic energy is converted into heat energy, or completely inelastic, in which case the objects stick together after the collision and the maximum amount of energy that is compatible with conservation of momentum is converted into heat energy.

In this investigation you will study both the conservation of kinetic energy and the conservation of momentum in an oblique collision between two objects.

Method 1

This method involves producing a collision between two balls, one stationary and one in motion. The only restriction on the balls you use is that the moving ball must be more massive than the stationary ball. (Otherwise the moving ball may rebound backwards, and the apparatus is not designed to record this fact.) In Investigation 1 you learned that objects projected horizontally take the same time to fall from the same height regardless of their horizontal velocity. This means that the horizontal displacement $\vec{d}$ of such a projected object while it falls is proportional to the object's horizontal velocity.

#007 Collision in Two Dimensions apparatus



Apparatus

Collision in Two Dimensions apparatus
carbon paper
large graph paper

Procedure

1. Measure and record the masses of the balls you intend to use. Release the incident ball several times from the same height on the ramp to measure the average distance it would travel if there were no collision. By doing this you can measure the velocity of the incident ball before the collision. You will also obtain a measure of how consistent the results are likely to be.
2. Move the target ball into position so that the centres of the two balls will be at the same height when the balls collide. Then release the incident ball from the same height as before, allowing it to collide head-on with the target ball. Be sure to catch both balls on their first bounce so that extra marks are not left on the paper. Label the marks left by the balls. Repeat this step several times to check the consistency of the results.
3. Swing the pivot point slightly off centre so that the target ball will be struck at an angle instead of head-on. Let the incident ball roll down the ramp from the same height as before to strike the target ball. Label the marks. Repeat this step several times to average your results.
4. Repeat Procedure 3, using a different angle of collision.
5. Measure the height of the target ball above the floor before the collision.

Data Analysis

1. Calculate how long in seconds it takes the balls to fall from the height of the desk.
2. Draw on the tracing paper the vector ($\vec{d}$ in diagram #007) that is proportional to the velocity of the incident ball before the collison.
3. Measure the length of this vector in centimetres, divide it by the time you calculated, and multiply it by the mass of the incident ball to obtain the magnitude of the momentum of the incident ball before the collision in g·cm/s. In order to represent this vector on a sheet of graph paper, you must scale it down to a convenient size. Decide on a scale factor which will enable you to represent this vector by a line about 25 cm long.

4. On your tracing paper, draw the momentum vector of each ball after each collision and measure the angle between each vector and the momentum vector before the collision.
5. Using your scale factor, make a vector diagram representing the *total* momentum of the two balls after each collision. (See Appendix 9.)
6. Calculate the total kinetic energy in joules before and after each collision.

Discussion of Results

1. Compare the total momentum before each collision to the total momentum after the collision. What is the percentage difference between their magnitudes? (See Appendix 5.) What is the angle between their directions?
2. Compare the total kinetic energy before each collision to the total kinetic energy after the collision. How elastic was the collision? (Convert the ratio of kinetic energy after to kinetic energy before into a percentage.)

Method 2

In this method the aim of the procedure is to produce a good strobe photo of an oblique collision between two discs on an air table. (See Appendix 11.)

Apparatus

strobe light
Polaroid camera
air table

Procedure

1. Mark the centre of each disc with a distinctive and visible symbol. Add a few small masses to one of the discs to make its mass different from that of the other disc. Record all pertinent measurements such as the mass and diameter of each disc, the flash rate of the strobe, the distance between the camera and the table, etc. The collision should occur

   (a) as close to the centre of the table as possible,

   (b) so that the direction of motion of either disc is changed by an angle between 45° and 135°, and

   (c) with the discs moving at different velocities.

2. Several trials will probably be necessary before you can meet all of the above requirements. When you are reasonably confident about the set-up, take a picture of the resulting collision. If the centre marks of the discs are too close together on the photo, slow down the flash rate of the strobe until you achieve good resolution.

3. Make a scale drawing of your results showing the positions of the disc centres before and after the collision (or duplicate the photo if possible). This will be your data sheet for later analysis.

Data Analysis

1. For each disc (or object) calculate the velocity before the collision and after. Be sure to multiply distances on the photo by the correct scale factor (as you did in Investigation 1) so that you calculate the actual velocities in cm/s.

2. Draw vector diagrams representing the total momentum before the collision and after. (See Appendix 9.) Be sure to use the correct angles when adding the momentum vectors.

3. Calculate the total kinetic energy in joules before and after the collision.

Discussion of Results

1. Compare the total momentum before the collision to the total momentum after the collision. What is the percentage difference between their magnitudes? (See Appendix 5.) What is the angle between their directions?

2. Compare the total kinetic energy before the collision to the total kinetic energy after the collision. How elastic was the collision? (Convert the ratio of kinetic energy after to kinetic energy before into a percentage.)

Method 3

In this method your data will be obtained from observing and analysing a film loop such as "Two-Dimensional Collisions—I & II" (available from the National Film Board) or "Totally Inelastic Auto Collisions" (available from Thornton & Associates). Follow the instructions given in each loop.

Data Analysis

1. For each disc (or object) calculate the velocity before and after the collision. If possible, multiply distances on the

photo by the correct scale factor (as you did in Investigation 1) so that you calculate the actual velocities in cm/s. If not, use the unit centimetres of photo per flash, where "flash" represents the period of the strobe light.

2. Draw vector diagrams representing the total momentum before the collision and after. (See Appendix 9.) Be sure to use the correct angles when adding the momentum vectors.
3. Calculate the total kinetic energy in joules before and after the collision. (Alternatively, use g • (cm/flash)^2 as a unit of energy.)

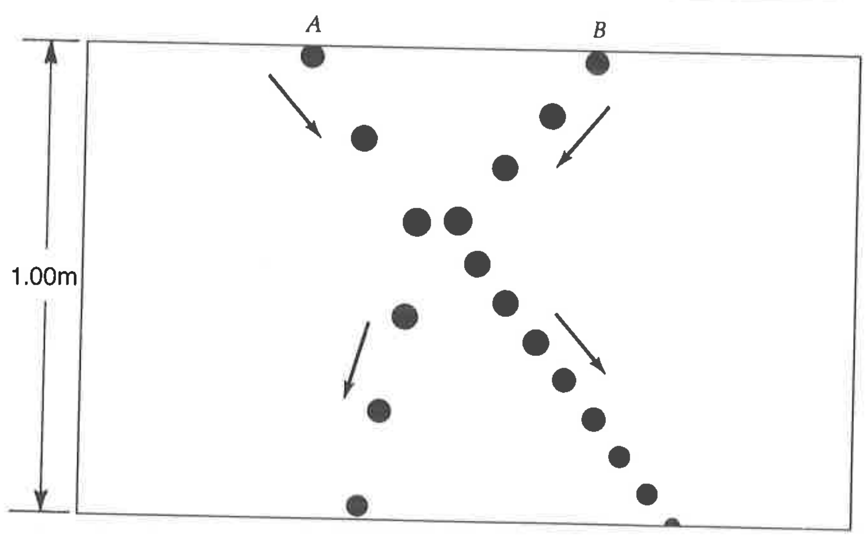
Discussion of Results

1. Compare the total momentum before the collision to the total momentum after the collision. What is the percentage difference between their magnitudes? (See Appendix 5.) What is the angle between their directions?
2. Compare the total kinetic energy before the collision to the total kinetic energy after the collision. How elastic was the collision? (Convert the ratio of kinetic energy after to kinetic energy before into a percentage.)

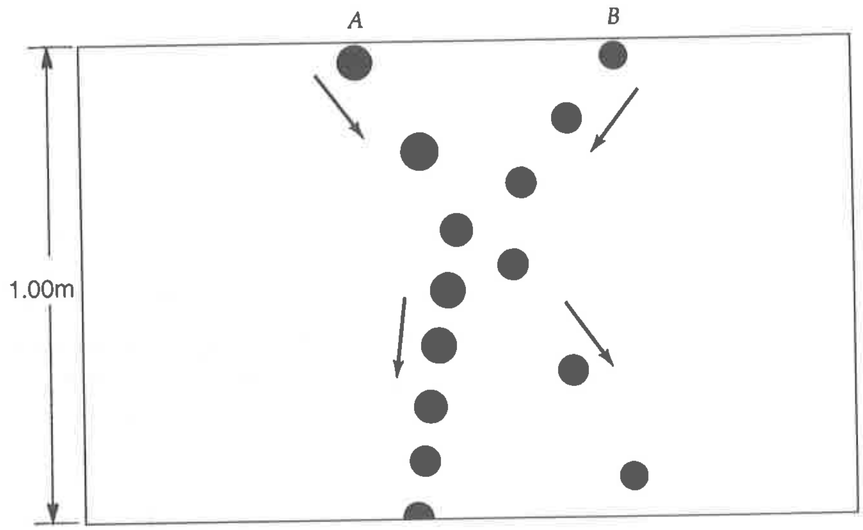
Method 4

The following illustrations show various oblique collisions between two objects. In illustration #008 the objects are of the same mass; in illustration #009 the objects are of different mass; and in illustration #010, the objects stick together after the collision.

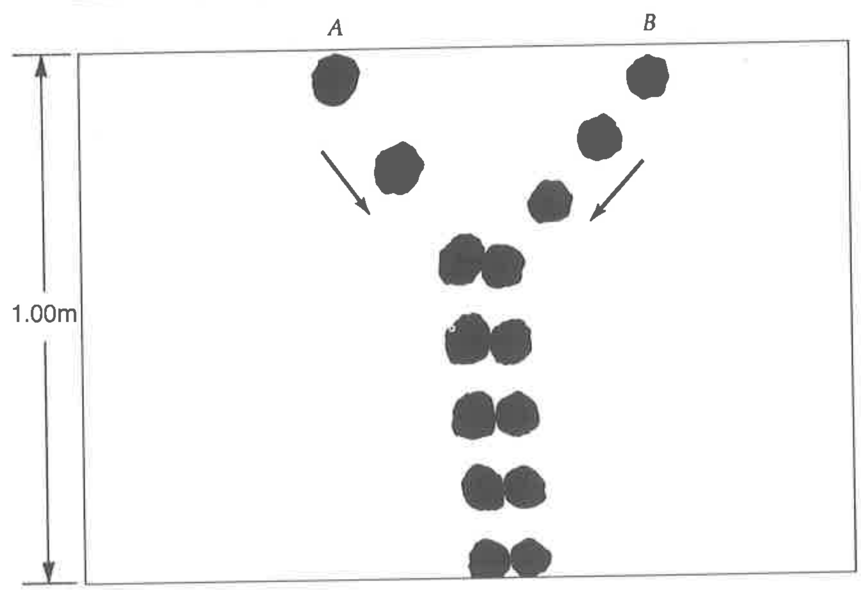
#008 m_a = 367 g, m_b = 367 g, strobe frequency = 20 Hz



#009 m_a = 539 g, m_b = 361 g, strobe frequency = 20 Hz



#010 m_a = 539 g, m_b = 361 g, strobe frequency = 20 Hz.



Data Analysis

1. For each disc (or object) calculate the velocity before and after the collision. Be sure to multiply distances on the photo by the correct scale factor (as you did in Investigation 1) so that you calculate the actual velocities in cm/s.

2. Draw vector diagrams representing the total momentum before the collision and after. (See Appendix 9.) Be sure to use the correct angles when adding the momentum vectors.
3. Calculate the total kinetic energy in joules before and after the collision.

Discussion of Results

1. Compare the total momentum before the collision to the total momentum after the collision. What is the percentage difference between their magnitudes? (See Appendix 5.) What is the angle between their directions?
2. Compare the total kinetic energy before the collision to the total kinetic energy after the collision. How elastic was the collision? (Convert the ratio of kinetic energy after to kinetic energy before into a percentage.)

All Methods

Regardless of the apparatus, procedure, method of analysing data, and discussion of results undertaken so far, complete this investigation as follows.

Additional Activities

1. The principle of conservation of momentum applies to explosions as well as collisions. Arrange two air pucks so that they are held together by a string with a small spring compressed between them. Record the "explosion" with a strobe camera when the string is cut (or burned through). Try this with the discs stationary at first, then with the discs in motion. Analyse the results.
2. If you have access to a pool table, try applying the Law of Conservation of Momentum to bank shots and to collisions of two or more balls. How does the rotation of the cue ball affect your results?

Practice Problems

1. A car of mass 3000 kg moving due South at 15 m/s collides completely inelastically with a second car of mass 2000 kg moving West at 12 m/s.
   (a) Draw a vector diagram representing the total momentum before the collision.
   (b) What is the total momentum after the collision (both magnitude and direction)?
   (c) What is the velocity of the combined vehicles after the collision?

2. A car moving North at 12 m/s strikes a stationary car of equal mass. The first car moves off after the collision at an angle of 30° East of North with a speed of 8.0 m/s.

   (a) What is the velocity of the struck car just after the collision?

   (b) Show that the collision is inelastic.

   (c) Explain how dents, skid marks, etc. show that kinetic energy has been lost.

   (d) If the collision were perfectly elastic what would the speeds of the cars be after the collision if the first car moved off at the same 30° angle?

3. A steel ball of mass 10 kg moves due East at 5.0 m/s. It collides with a rubber ball of mass 5.0 kg moving at 10 m/s due North. After the collision the steel ball moves at an angle of 60° East of North with a speed of 4.0 m/s. What is the velocity of the rubber ball after the collision?

4. A ball of mass 3.0 kg moving at a speed of 3.0 m/s has a head-on collision with a stationary ball of mass 4.0 kg.

   (a) If the collision were completely inelastic what would be the speeds of the two balls after the collision?

   (b) If the collision were perfectly elastic what would be the speeds of the two balls after the collision?

5. A child's ball, of mass 250 g, rolls due East along the ground at 4.2 m/s towards a stationary bowling ball, of mass 3.2 kg. After the collision, the bowling ball travels in a direction 32° South of East at 38.7 cm/s.

   (a) What is the speed of the child's ball after the collision?

   (b) What is the direction in which the child's ball travels after the collision?

   (c) What fraction of the original kinetic energy is transformed to heat energy?

6. A bowling ball, of mass 4.00 kg, strikes a bowling pin of mass 110.0 g. obliquely. Before the collision, the ball was travelling at 3.10 m/s down the alley; after the collision it travels at 2.99 m/s and its direction has changed by 1.3°.

   (a) Calculate the speed of the pin immediately after the collision.

   (b) In what direction does the pin travel immediately after the collision?

   (c) What fraction of the ball's kinetic energy is transferred to the pin?

7. A stationary car is struck obliquely by another car of equal mass. Immediately after the collision, the car that was originally stationary travels due East at a speed of 12 km/h; the other travels 66° North of East at a speed of 24 km/h. What was the original velocity of the incident car?

8. A football player of mass 95 kg running North with a speed of 5.6 m/s collides obliquely with another player of mass 147 kg running South at 5.6 m/s. After the collision, the direction of the more massive player is due East and his speed is 3.24 m/s. What is the velocity of the lighter player?

9. Partners in mixed doubles at tennis collide: the girl, of mass 51 kg, running due Northeast at 3.20 m/s and the man, of mass 89 kg, running due Northwest at 5.30 m/s. Immediately after the collision, the man's direction is 89° North of East and his speed is 3.75 m/s. What are the speed and direction of the girl? Was the collision completely elastic?

10. Two billiard balls of equal mass collide obliquely on a billiard table: one travelling 30.0° North of East at 3.2 m/s, the other travelling due West at 4.6 m/s. After the collision, the ball that was originally travelling due West travels due East at 2.77 m/s.

    (a) What are the speed and the direction of the other?

    (b) Show whether or not the collision is elastic.

11. An air puck of mass 2.00 kg moving at 1.60 m/s 80.0° North of East collides obliquely with another air puck of mass 1.00 kg. After the collision, the 2.00-kg puck travels 3.3° West of North at 1.64 m/s and the 1.00-kg puck travels 62° North of East at 0.596 m/s.

    (a) What were the speed and the direction of the 1.0-kg puck before the collision?

    (b) Calculate the total kinetic energy of the system before and after the collision. Was the collision elastic?

12. Two pucks on an air table collide: the first, of mass 1.5 kg, is travelling 85° North of East and the other, of mass 2.5 kg, is travelling due Northwest. After the collision, the first travels 43° North of West at 1.75 m/s; the second travels 78° North of West at 1.09 m/s. Find the speeds of the two pucks before the collision.

Note to Teachers: For general solutions to problems involving collisions in two dimensions and for help in making up such problems, see Appendix 12.

Science is not just the business of collecting facts or stating laws or directing experiments. It is above all an act of sensing the best choice of view or the most fruitful line of investigation of a growing understanding of nature.

Eric M. Rodgers

Equilibrium

Investigation 3:

Equilibrium of Forces

When forces act on an object without producing motion of any kind, both the object and the forces are said to be in **static equilibrium.**

In this investigation you will study forces which act at a single point on a stationary object. Two different methods can be used.

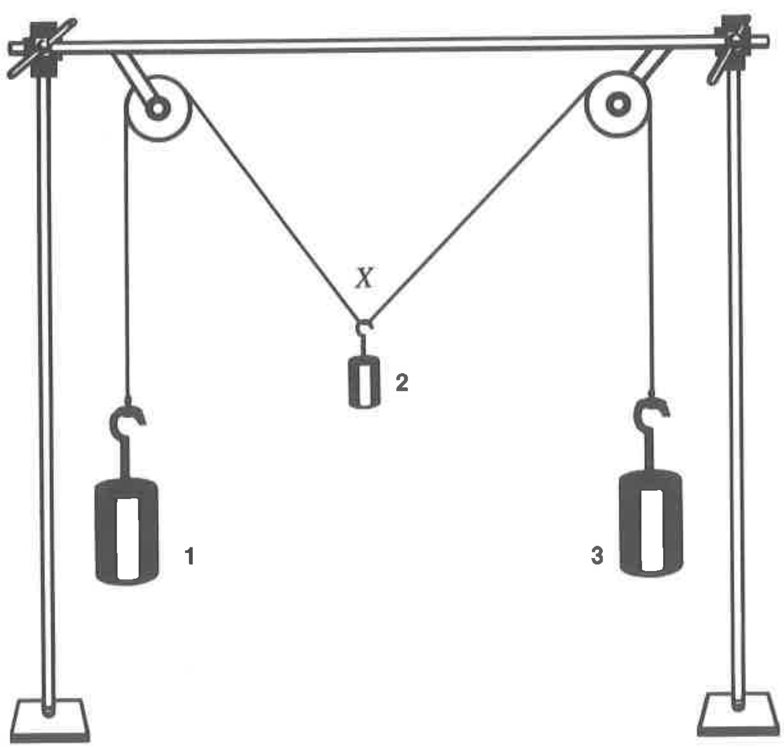
Method 1

This method uses the force of gravity on various masses.

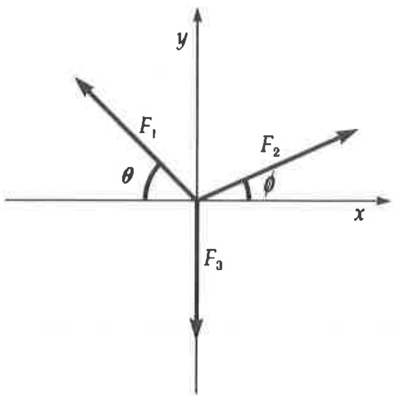
Apparatus

2 support stands
3 different hooked masses
2 pulleys
string

#011 Pulleys and hooked masses suspended from support stands



#012 Lines of action for the three forces



Procedure

1. Suspend two pulleys from the support stands (figure #011).
2. With a long string, suspend two different masses from the pulleys. (Hold the string so that they do not fall.)
3. Hook a third mass near the middle of the string and slowly release it. If it does not come to equilibrium, use a lighter (or heavier) mass.
4. Three forces are now acting on point *X:* the forces of gravity on the three masses. Place a sheet of paper behind the point *X.* Use two dots to mark the line of action of each force (figure #012). Record the three masses on your chart. Multiply each mass by 9.8 N/kg to find the force of gravity on that mass.
5. Repeat the procedure using a different set of masses.

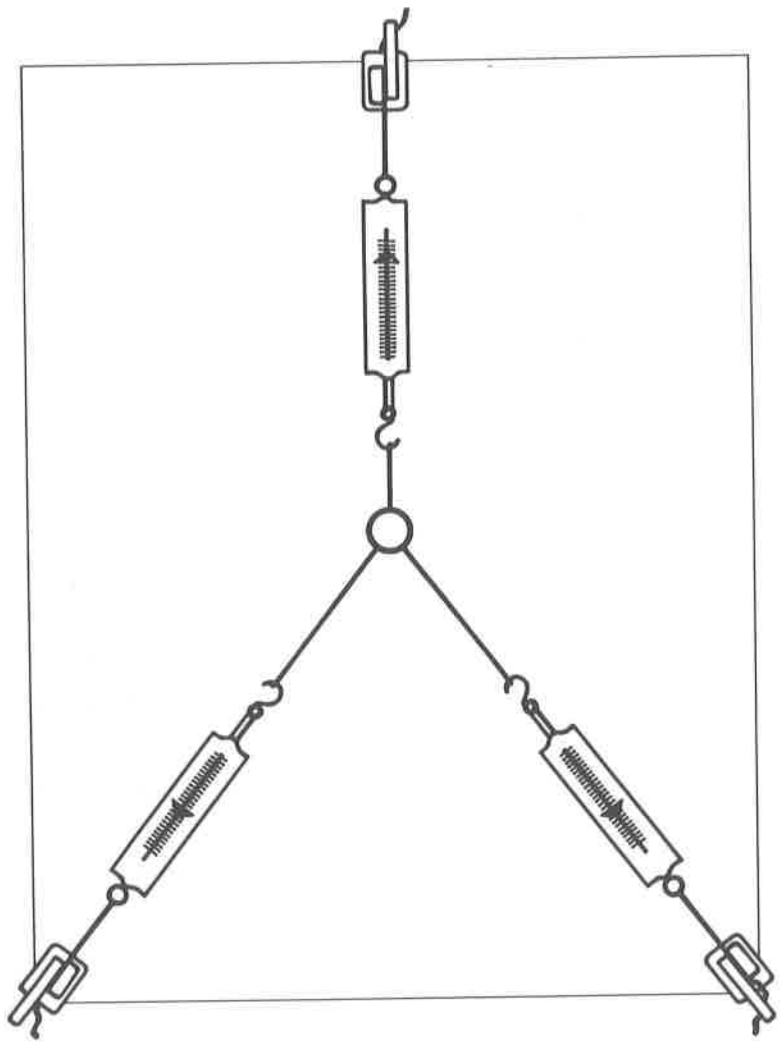
Method 2

This method involves the use of spring balances attached to the edges of a horizontal table or board.

Apparatus

3 spring scales
small ring
string

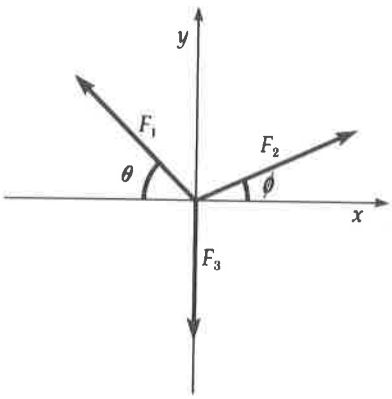
#013 Spring balances attached to a small ring



Procedure

1. Adjust each spring scale so that it reads zero when placed horizontally.
2. Attach each spring scale to a small ring by a short loop of string.
3. Pull on each spring and adjust their positions until they have readings within 5 N of each other.

#014 Lines of action of three forces



4. There are now three forces acting on the ring (which is small enough to be considered a point). Place a sheet of paper beneath the ring. Use dots to mark the line of action of each of the forces (figure #014). Record the actual values of the forces on your chart.

5. Repeat the procedure for different positions of the spring scales.

All Methods

Regardless of the apparatus and procedure used, complete the investigation as follows.

Data Analysis

For each force diagram do the following:

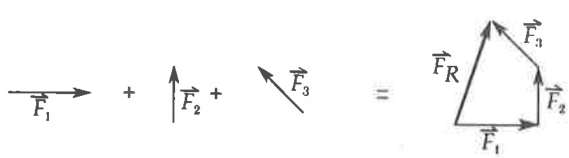
1. Use a suitable scale to extend the lines on your diagram until each one is of the right length to represent the force acting in that direction.

2. Measure the angles between the forces as shown in figure #018.

3. Make a scale drawing showing the vector addition of $\vec{F}_1$, $\vec{F}_2$, and $\vec{F}_3$ (be sure to use the correct angles between the forces). (See Appendix 9.)

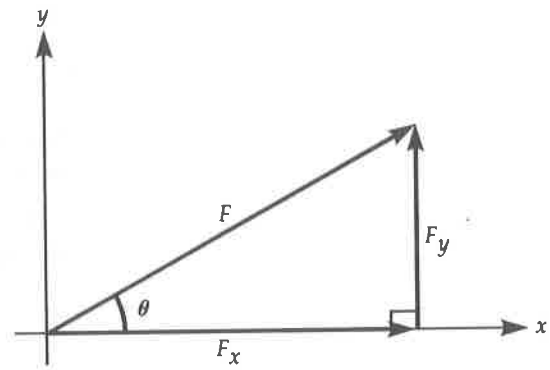
   Vectors are easily added using the tip-to-tail method (see illustration #015):

#015 Adding vectors



4. Any vector can be represented as the sum of two other vectors, its *components*. These vectors are usually chosen to be perpendicular to each other (see illustration #016):

#016 Components of vectors



$$F_x = F\cos\theta, \quad F_y = F\sin\theta,$$
$$F_x^2 + F_y^2 = F^2$$

$\vec{F}_x$ is called the x component of the force $\vec{F}$.
$\vec{F}_y$ is called the y component of the force $\vec{F}$.

Calculate the components of each of the forces (using these equations). (See Appendix 9.)

$$F_{1x} = -F_1\cos\theta \qquad F_{1y} = F_1\sin\theta$$
$$F_{2x} = F_2\cos\phi \qquad F_{2y} = F_2\sin\phi$$
$$F_{3x} = 0 \qquad F_{3y} = -F_3$$

The − signs are included to indicate the directions.

5. Calculate $\Sigma\vec{F}_x$ and $\Sigma\vec{F}_y$.

Discussion of Results

1. Did the three vector forces add to zero? If not, what did they add to? Express the magnitude of this vector as a percentage of the average magnitude of the three forces used.
2. Were the sum of the x-components and the sum of the y-components of the forces equal to zero? If not, express the magnitude of each sum as a percentage of the average magnitude of the three forces used.
3. Try to explain how error could occur in this experiment. Friction is always present, of course; how would it affect your results? What other sources of error are present when using your particular apparatus and procedure?

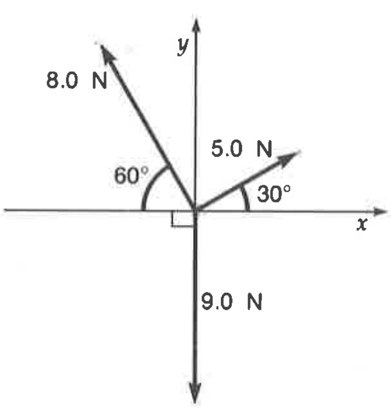
4. Are the vector sum of the forces and the vector sums of the x-components and the y-components equal to zero, within experimental error?

Additional Activities

1. Combine your apparatus with that of another group and try to achieve equilibrium with 5 or 6 different forces (all in the same plane).
2. Use 3 or 4 spring balances to lift a 1-kg mass. First, lift directly upwards, then lift at various angles to the vertical. Measure the angles for each spring and calculate the vertical component of each force. Check if the sum of the vertical components is equal to the force of gravity on the mass.

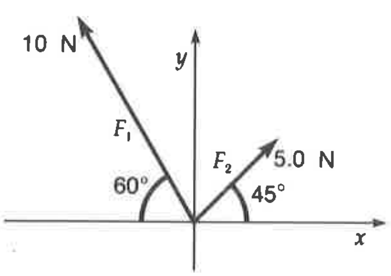
Practice Problems

#017 Diagram for Problem 1



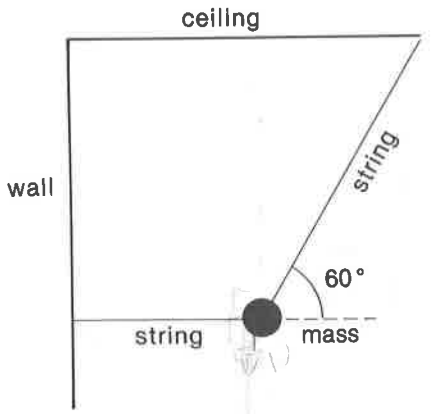
1. Make an accurate vector diagram of the forces shown in diagram #017 to check if the forces are in equilibrium.
2. Check your result in question 1 by calculating the vector components and finding $\Sigma\vec{F}_x$ and $\Sigma\vec{F}_y$.

#018 Diagram for Problem 3



3. (a) Make an accurate vector diagram of the forces shown in diagram #018 to calculate the force necessary to produce equilibrium with $\vec{F}_1$ and $\vec{F}_2$.
   (b) Calculate the resultant of $\vec{F}_1$ and $\vec{F}_2$ using components.
   (c) Find the x and y components of the force necessary to produce equilibrium with $\vec{F}_1$ and $\vec{F}_2$.
   (d) What is the magnitude and direction of the force needed in (c)? (This force is called the equilibrant of $\vec{F}_1$ and $\vec{F}_2$.)

#019 Diagram for Problem 4



4. An object on which the force of gravity is 50 N is supported by two strings as shown in diagram #019. Calculate the magnitude of the force of tension exerted by each string on the object.

5. A boy on whom the force of gravity is 400 N hangs on to the middle of a rope stretched between two trees. The rope sags in such a way that it makes an angle of 170° at the boy's hands. What force does the rope exert on each tree?

When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of Science.

Lord Kelvin

Investigation 4:

Equilibrium of a Loaded Beam

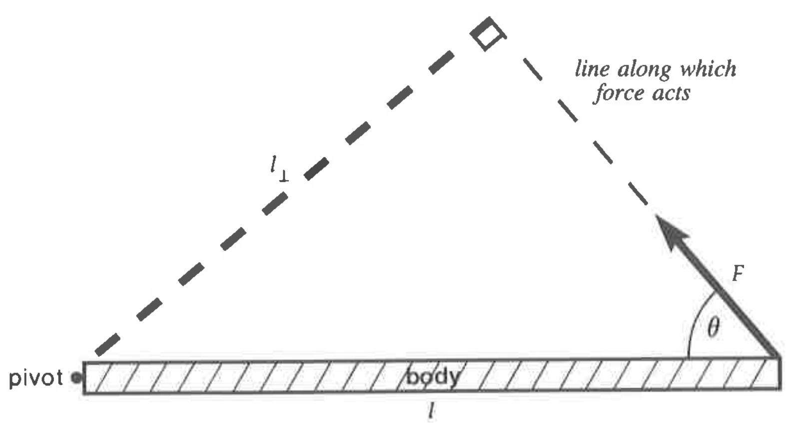
In the previous investigation, you studied forces that acted at the same point on an object. In this investigation, you will study forces that act at different points. When forces act at different points on an object, their vector sum must be zero for the object to be in *translational* equilibrium. (An object in translational equilibrium is not travelling bodily from place to place.) However, such forces could cause the object to rotate. For an object to be in static equilibrium (completely stationary), it must also be in *rotational* equilibrium.

In this investigation you will study forces applied at different points on an object that is in static equilibrium; that is, rotational as well as translational equilibrium.

When a force causes an object to rotate, it rotates about a pivot. The turning effect of a force on the object is called the *torque* of that force about the pivot.

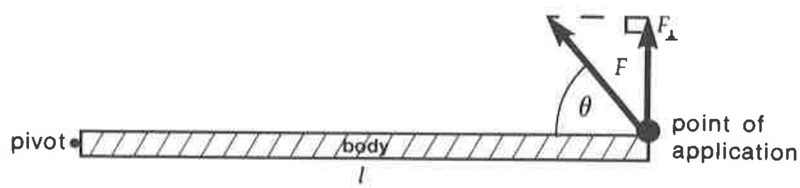
The torque of a force about a pivot is equal to the product of the magnitude of the force (F) and the perpendicular distance ($l_\perp$) from the pivot to the line along which the force acts. For figure #020, $T = l_\perp F$.

#020 $T = l_\perp F$



Another equation that can be used to describe the torque is $T = lF_{\perp}$, where $F_{\perp}$ is the component of the force perpendicular to the line from the pivot to the point of application of the force.

#021 $T = lF_{\perp}$

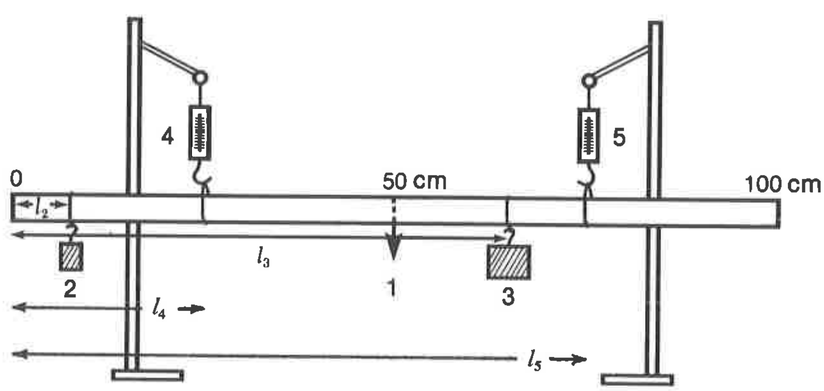


Deciding which equation to use is a matter of convenience, as they both give the same result, $Fl \sin \theta$. Torques which cause an object to rotate clockwise can be considered as negative, while those which cause counter-clockwise rotation can be considered to be positive.

Apparatus

2 spring balances
metre stick (beam)
2 support stands
string
hooked masses
triple-beam balance

#022 Assume that there is a pivot located at the 0 end of the beam. The distances of the points of application of the forces are to be measured from this pivot. Keep in mind that the force of gravity on a uniform beam can be considered to act at its centre.



Procedure

1. Weigh the beam and record the force of gravity on it (F_1) in newtons.
2. Support the metre stick by using the spring balances attached to the support stands. The stick should be horizontal and the balances vertical. Remember to adjust the balances to zero before attaching them to the beam.
3. Add two different masses to the beam and adjust their distances from the pivot (l_2, l_3) until the beam is horizontal. The masses should not be directly under the balances. Calculate the force of gravity on each mass by multiplying its mass by 9.8 N/kg.
4. Record all of the forces and their distances from the pivot.
5. Support the beam with the spring balances at an angle to the beam and repeat the original procedure. Be sure to measure the angles between each spring's line of action and the beam.

Data Analysis

1. Calculate $\Sigma\vec{F}$.
2. Calculate ΣT.

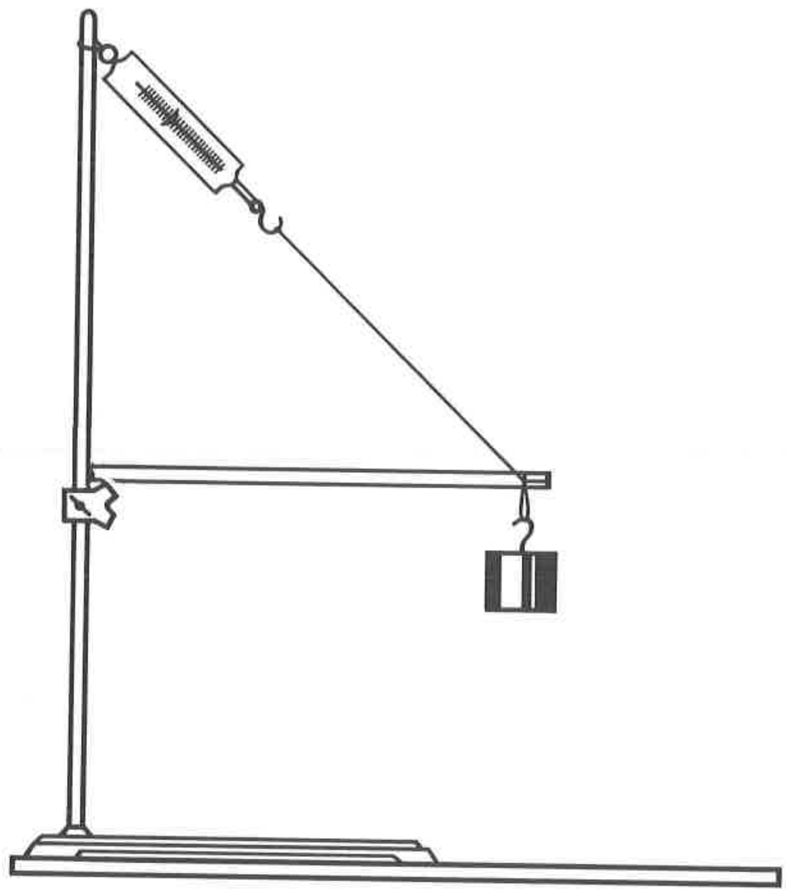
Discussion of Results

1. Express $\Sigma\vec{F}$ as a percentage of the average magnitude of the forces.
2. Express ΣT as a percentage of the average magnitude of the torques.
3. Recalculate the torques assuming that the pivot is at the centre of the beam (i.e., measure all distances to the forces from the centre of the beam). Show all the steps of your calculation. Do your results differ significantly from those of the previous calculation? Explain any differences found.

Additional Activities

1. Repeat the investigation using at least four different masses suspended from four different points on the beam.
2. Support the beam and one mass by one spring balance as shown in figure #023, making sure that the beam is only resting on the support at the stand and is not rigidly clamped to prevent it from rotating. (This is known as a cantilever support.) Carefully measure the angle between the beam and the line of action of the spring.
   Calculate $\Sigma\vec{F}$ and ΣT.

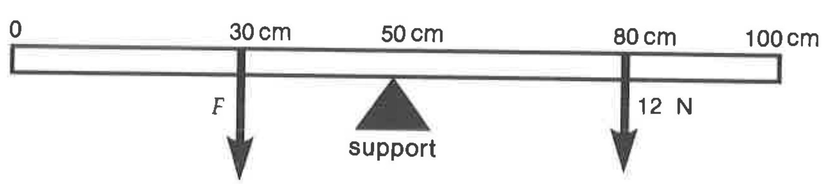
#023 Cantilever support



Practice Problems

1. The beam shown in diagram #024 is in static equilibrium (i.e., translational and rotational equilibrium).

#024 Diagram for Problem 1

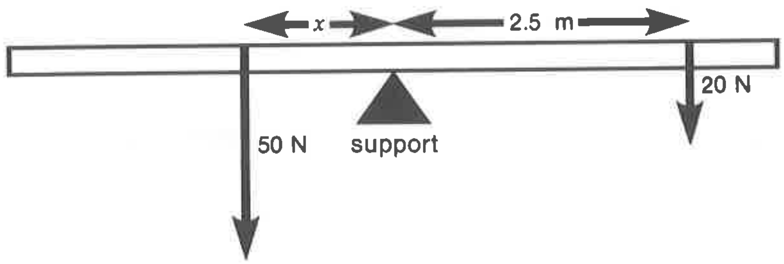


(a) What is the magnitude of the force $\vec{F}$?

(b) If the force was moved to the 10 cm point what would its magnitude have to be to keep the beam in equilibrium?

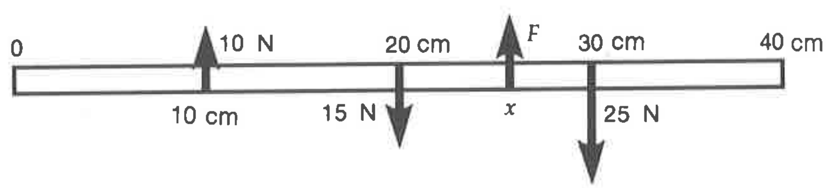
2. The beam shown in diagram #025 is in static equilibrium (i.e., translational and rotational equilibrium). What is the distance to the 50 N force? (Assume that the beam is uniform.)

#025 Diagram for Problem 2



3. For the situation shown,

#026 Diagram for Problem 3



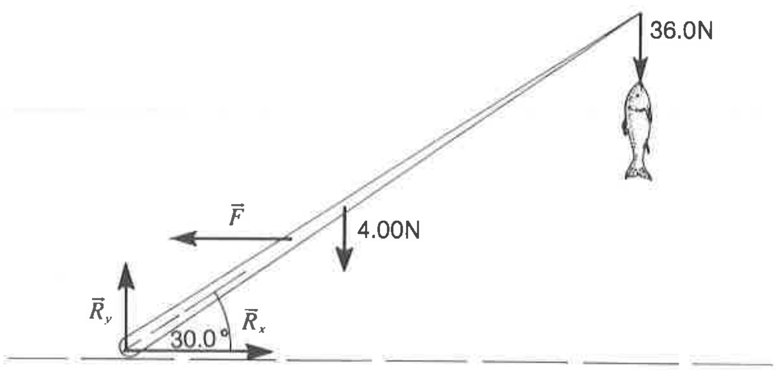
(a) find the Force (F) needed to produce translational equilibrium.

(b) find the distance x needed to produce static equilibrium.

4. (a) A fishing rod 3.00 m long is held at 30.0° to the horizontal by a horizontal force F exerted at a point 1.10 m from the pivot end of the rod. What force is required if the force of gravity on the fish is 36.0 N? Assume that the force of gravity on the rod (4.00 N) acts at a point 1.20 m from the pivot.

   (b) What are the magnitude and the direction of the resultant reaction force $\vec{R}$ at the pivot point?

#027 Diagram for Problem 4



It is important to realize that in physics today, we have no knowledge of what energy is.

Richard P. Feynman

Circular Motion and Gravitation

Investigation 5:

Circular Motion

When an object moves at a constant speed in a circular orbit its velocity vector is constantly changing direction, always being deflected towards the centre of the circle. Therefore we say that the object is continuously accelerating. If the circular orbit has a radius of r and the object completes one orbit in a time of T, then the speed v of the object is

$$v = \frac{2\pi r}{T}$$

and the magnitude of the object's acceleration is

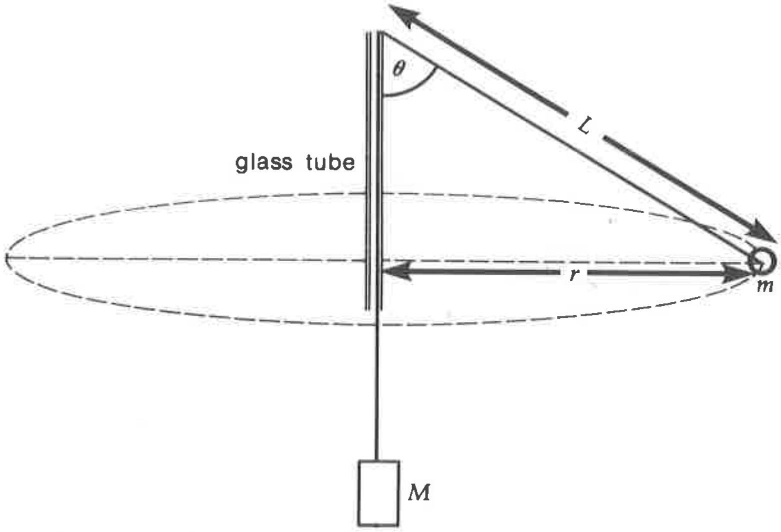
$$a_c = \frac{v^2}{r} \text{ or } a_c = \frac{4\pi^2 r}{T^2}$$

The direction of the acceleration is towards the centre of the circle; that is why the acceleration is called a **centripetal** acceleration.

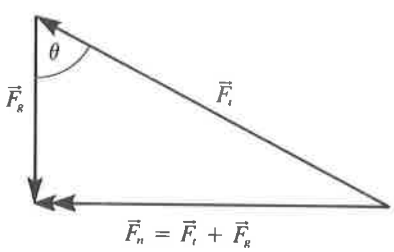
This acceleration, like any other, is caused by an unbalanced force, the centripetal force, which must have a magnitude of $F_c = ma_c$, where m is the mass of the object. This unbalanced force can be either a gravitational force (as in the case of a satellite orbiting the Earth), an electrical force (as in the case of electrons orbiting a nucleus), or a mechanical force (as when a string is tied to an object being whirled in a circle).

This last type of centripetal force is the easiest to study and will therefore be the one considered in this investigation. The diagrams #028 and #029 illustrate the actual situation.

#028 Circular motion apparatus



#029 The forces on the revolving mass are the force of gravity $\vec{F}_g$ and the force of tension $\vec{F}_t$ from the string.



The force of tension in the string is Mg, the force of gravity on the central mass. The upward component of this force, $Mg \cos \theta$, balances the force of gravity mg on the smaller mass

$$Mg \cos \theta = mg$$

so $$\cos \theta = \frac{m}{M}$$

The horizontal component of the force of tension in the string, $Mg \sin \theta$, is unbalanced. This is the centripetal force, the force that keeps the smaller mass from flying out of its orbit.

$$Mg \sin \theta = F_c = ma_c = m\frac{4\pi^2 r}{T^2}$$

Now, from figure #028, $r = L \sin \theta$,

so $$Mg \sin \theta = \frac{m\, 4\pi^2 L \sin \theta}{T^2}$$

or $$Mg = \frac{m\, 4\pi^2 L}{T^2}$$

or $$T^2 = \frac{m\, 4\pi^2 L}{Mg}$$

According to this theoretical equation, the square of the period of the orbit, T^2, is directly proportional to the length L of the string and the mass m of the orbiting object and inversely proportional to the mass of the central object (if all the other variables are kept constant). These predictions can be checked experimentally.

Apparatus

glass rod, fire polished both ends (15 cm length)
150 cm waxed dental floss or nylon monofilament line (#5)
hooked weights
triple-beam balance
stopwatch
metre stick

Procedure

1. Securely tie the string to a 50-g mass. Thread the other end of the string through the glass rod and tie it securely to another mass about 3 times as heavy. Record both masses.
2. Using the centre of the 50-g mass as the zero point, mark the string at intervals of 20 cm to about 120 cm.
3. Holding the glass rod firmly, swing the smaller mass around in a circle until a stable orbit is achieved with the 20 cm mark at the top edge of the tube. Keep the motion of the tube itself to a minimum. Have your partner time 10 complete revolutions with the stopwatch so that the period of revolution can be accurately measured.
4. Repeat the procedure for a longer length of string. Make a table showing the period of revolution, T, and the length of string, L.
5. Repeat Procedures 3 and 4, this time keeping L constant at about 50 cm and varying m, always keeping it less than M. Record L, M, and the value of T for each value of m.
6. Repeat Procedures 3 and 4 again, this time keeping L constant at about 50 cm and m constant at about 75 g, but varying M, keeping it greater than m. Record L, m, and the value of T for each value of M.

Data Analysis

1. Graph T^2 vs. L, T^2 vs. m, and T^2 vs. $1/M$. (See Appendix 3.) Write the equation of each reasonably straight line. (See Appendix 4.)

Discussion of Results

1. Compare the actual equation of each graph with the theoretical equation. (See Appendix 6.)
2. Explain why m must always be less than M. What would happen mathematically if it were not? What would happen physically?

3. The friction of the string on the tube is one definite source of error in this investigation. List several ways of reducing this error, or describe an alternative technique that would be more friction-free. How would you expect this friction to affect your graphs?
4. List other sources of error that might affect your results.
5. Have you proved that

$$F_c = m\,\frac{4\pi^2 r}{T^2}$$

within experimental error?

Additional Activities

1. Use a piano stool (or some similar seat) that rotates easily. Have someone spin you in a circle while your arms and legs are stretched out. Slowly pull in your arms and legs as far as you can and observe the effect on your rate of spin. Explain your observations in terms of the law of conservation of angular momentum.
2. Roll a solid cylinder and a ring of the same diameter and mass down an incline. Which one has the greater speed at the bottom of the incline? Try to determine the factors that affect the results, such as the mass of each and the diameter of each. Explain your results using the concept of rotational inertia.

Practice Problems

1. A ball of mass 0.50 kg is swung in a circle of radius 2.0 m with a period of 1.5 s (as in figure #028).

   (a) What is the speed of the ball?

   (b) What is the acceleration of the ball?

   (c) What centripetal force must be exerted by the string that keeps the ball in orbit?

   (d) What is the mass of the central object?

   (e) What angle does the string make with the vertical?

   (f) How long is the string, L?

2. A record of diameter 30 cm rotates on a turntable at 33.3 r/min.

   (a) How fast is the outside edge of the record moving?

   (b) How many times as fast would it move if the frequency were raised to 78 r/min?

3. A satellite is kept in a circular orbit 300 km above the surface of the Earth by the force of gravity. At this altitude the acceleration due to gravity is only 8.9 m/s^2. The radius of the Earth is 6.4×10^6 m.

   (a) Calculate the period of the satellite.

   (b) Calculate the speed of the satellite.

4. Calculate the frequency with which the Earth would have to rotate so that an object on the surface of the Earth at the equator would just become "weightless" (*all* of the gravitational force on it would be necessary to keep the object in its "orbit" as the Earth rotated).

Science is built of facts the way a house is built of bricks, but an accumulation of facts is no more science than a pile of bricks is a house.

Henri Poincaré

Investigation 6:

Elliptical Orbits

In the previous investigation you studied the motion of an object travelling in a circle. Most satellites of the Sun and the Earth, however, do not move in circular orbits. Johannes Kepler (1571–1630) formulated his laws of planetary motion from the extensive observations of Tycho Brahe (1546–1601).

First Law: Each planet moves in an elliptical orbit with the Sun at one focus of the ellipse.

Second Law: Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

Third Law: The ratio of the cube of a planet's average distance from the Sun to the square of its period of revolutions is a constant ($R^3/T^2 = k$).

Sir Isaac Newton (1642–1727) showed that Kepler's Laws followed from his own Law of Gravitation, $F = GmM/R^2$, where F is the gravitational force exerted by the Sun (of mass M) on a planet (of mass m) a distance R away, and G is a universal constant.

In this investigation, you will see that Newton's Laws and Kepler's Laws are related. Several different methods are possible, depending on the facilities of your laboratory.

Method 1

This method uses modern planetary data to demonstrate Kepler's Third Law and Newton's Law of Gravitation.

#030 Planetary data table

Planet	Radius (m)	Period (s)
Mercury	5.79×10^{10}	7.60×10^{6}
Venus	1.08×10^{11}	1.94×10^{7}
Earth	1.49×10^{11}	3.16×10^{7}
Mars	2.28×10^{11}	5.94×10^{7}
Jupiter	7.78×10^{11}	3.74×10^{8}
Saturn	1.43×10^{12}	9.30×10^{8}
Uranus	2.87×10^{12}	2.66×10^{9}
Neptune	4.50×10^{12}	5.20×10^{9}
Pluto	5.9×10^{12}	7.82×10^{9}

Data Analysis

1. To find out whether R^3 is really proportional to T^2, as Kepler found, you should plot R^3 against T^2 to see if you obtain a straight line through the origin. However, the values of R^3 and T^2 span six orders of magnitude, so it is very difficult to plot them on one graph. Instead, we plot log R against log T.

   If Kepler's Third Law is true,

$$\frac{R^3}{T^2} = k$$

$$R^3 = kT^2$$

$$3 \log R = \log k + 2 \log T$$

$$\log R = \frac{1}{3} \log k + \frac{2}{3} \log T$$

$$\log R = \frac{2}{3} \log T + \frac{1}{3} \log k.$$

Then a graph of log R against log T will have a slope of 2/3 and a y-intercept equal to 1/3 log k.

Look up log R and log T for each planet and plot a graph of log R vs. log T. (The logarithms have no units.) (See Appendix 3.) If this graph is straight, write its equation. (See Appendix 4.)

2. If Newton's Law of Gravitation is true,

$$F = \frac{GmM}{R^2}$$

$$\frac{F}{m} = \frac{GM}{R^2}$$

Since the gravitational force F on a planet is unbalanced,

$$\frac{F}{m} = a$$

where a is the planet's acceleration. Hence

$$a = \frac{GM}{R^2}.$$

Now G is a universal constant and M is the mass of the Sun, so GM is the same for all planets. This equation says, therefore, that a is inversely proportional to R^2. Now the gravitational force on a planet is directed at right angles to the planet's velocity (approximately), so the planet's acceleration is centripetal, and

$$a_c = \frac{4\pi^2 R}{T^2}.$$

Calculate a_c for each planet.

3. Again, we plot a log-log graph because of the range of the data. If

$$a_c = \frac{GM}{R^2}$$

$$\log a_c = \log (GM) - \log R^2$$

$$\log a_c = -2 \log R + \log (GM)$$

If Newton's Law of Gravitation is true, a graph of log a_c against log R will have a slope of -2 and a y-intercept equal to log (GM).

Calculate log a_c for each planet and plot a graph of log a_c vs. log R. (See Appendix 3.) If it is straight, write its equation. (See Appendix 4.)

Data Interpretation

1. Is Kepler's Third Law true? Back up your answer by using your graph. Find the value of k in m^3/s^2 from the y-intercept of the graph.
2. Is Newton's Law of Gravitation true? Back up your answer by using your graph. Find the value of GM from the y-intercept of your graph.
3. Using the fact that the gravitational force, as given by Newton's Law of Gravitation, is a centripetal force for planets of the Sun, show that Kepler's Third Law follows. How is k $(= R^3/T^2)$ related to the mass of the Sun? Given that $G = 6.67 \times 10^{-11}$ $N \cdot m^2/kg^2$, calculate the mass of the Sun, using the value of k from your graph. Do you expect k to have the same value for satellites of the Earth? Explain.

Method 2

This method uses Kepler's First and Second Laws to determine the nature of the force holding satellites of the Earth in orbit. Diagram #031 shows the positions of a satellite in an elliptical orbit at equal time intervals. These positions were determined by using Kepler's Second Law.

Procedure

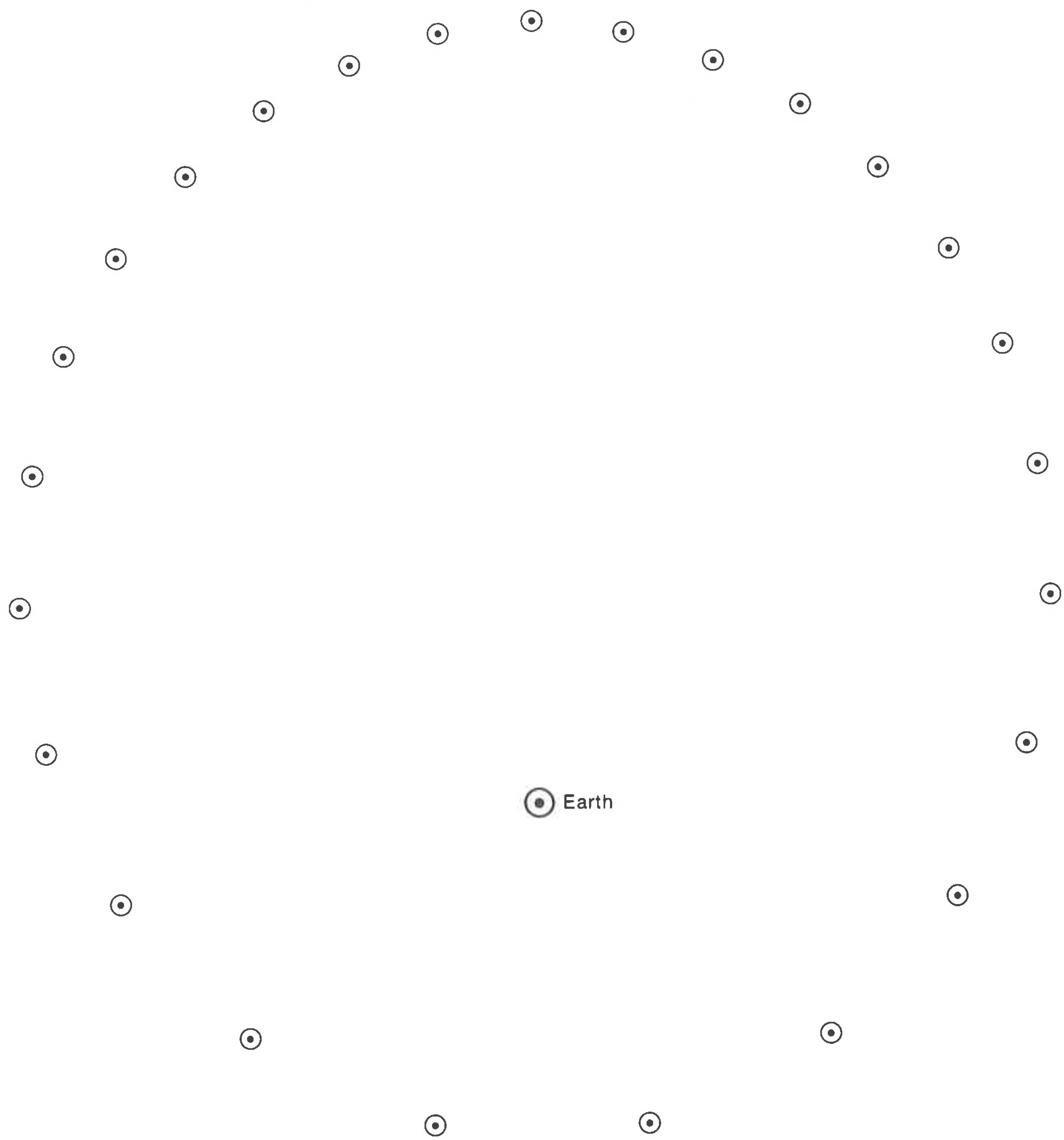
1. Place a sheet of paper over diagram #031. Mark the focus and the centre of each position of the satellite. Number these positions.

Data Analysis

1. First you should check that this diagram is an accurate illustration of a stroboscopic photograph of the motion of a satellite; that is, check that if you connect each image of the satellite to the Earth with a straight line, the area between successive lines is always the same.

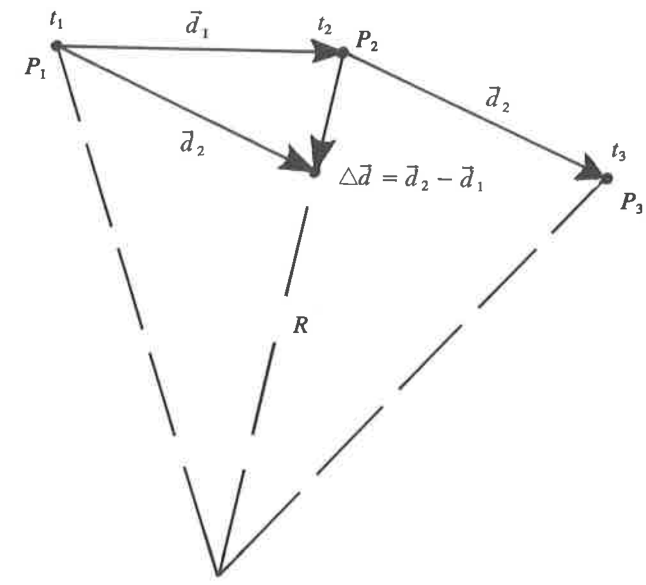
   Join each image of the satellite to the Earth and to the adjacent images. Measure the length of each line joining the images to the Earth. Then, by using one of the sides of each triangle as the base, measure the length of the corresponding height of the triangle. (The area of the triangle is not exactly the area swept out by the line joining the satellite to the Earth, because the satellite's path is curved, but it is approximate.) From these measurements, calculate the area of each triangle. (See Appendix 10.)

#031 Positions of a satellite in an elliptical orbit shown at equal time intervals



2. Next, you should determine the approximate acceleration of the satellite in each position as shown in figure #032. Accurately measure $\Delta\vec{d}$ for each position. (The vector shown is $\Delta\vec{d}$ for P_2.) Make a table of $\Delta\vec{d}$ and the corresponding value of R for each position.

#032 Variables to be measured for each position of the satellite



Data Analysis

The average velocity of the satellite between P_1 and P_2 is

$$\frac{\vec{d}_1}{\Delta t}$$

where Δt is the time interval between t_1 and t_2. Now the satellite is accelerating continuously, and its acceleration is not constant; the acceleration itself is continuously changing in magnitude and direction. However, over a short interval like Δt we can assume that it does not change very much; it is approximately constant. In that case the satellite's average velocity between t_1 and t_2 is approximately equal to its instantaneous velocity at the midpoint between t_1 and t_2.

Similarly the instantaneous velocity at the midpoint in time between t_2 and t_3 is approximately

$$\frac{\vec{d}_2}{\Delta t}.$$

Now $\triangle t$ is the same for both; therefore the instantaneous velocity midway in time between two successive positions of the satellite is proportional to the displacement vector drawn between those two positions.

If we subtract these two vectors, the first from the second, we will have a vector $\triangle\vec{d}$ which is proportional to the change in velocity during the time interval from the midpoint of t_1 and t_2 to the midpoint of t_2 and t_3. The length of this time interval is $\triangle t$. If we divide $\triangle\vec{d}$ by $\triangle t$, we will have a vector which is proportional to the average acceleration between the midpoint of t_1 and t_2 and the midpoint of t_2 and t_3. The length of the time interval will always be $\triangle t$, no matter what points on the ellipse we are working with, so the vector $\triangle\vec{d}$ is (approximately) proportional to the average acceleration of the satellite between the midpoint of t_1 and t_2 and the midpoint of t_2 and t_3.

Now, as we have said, the acceleration of the satellite is continuously changing. If we assume that it changes at a fairly steady rate over a short time interval like $\triangle t$, then the average acceleration over this time interval is equal to the instantaneous acceleration at the midpoint of the time interval. The point midway between the midpoint of t_1 and t_2 and the midpoint of t_2 and t_3 is t_2.

The vector $\triangle\vec{d}$, therefore, is (approximately) proportional to the acceleration $\vec{a}$ of the satellite at t_2.

$$\triangle\vec{d} \propto \vec{a}$$

The unbalanced gravitational force $\vec{F}$ on the satellite from the Earth is what causes this acceleration, so

$$\vec{F} = m\vec{a}$$

where m is the mass of the satellite. This mass is constant, however, so

$$\vec{F} \propto \vec{a}.$$

Consequently

$$\vec{F} \propto \triangle\vec{d}.$$

Plot $\triangle d$ versus R to see how the gravitational force F on the satellite varies with R.

Discussion of Results

1. In which direction do the $\triangle\vec{d}$ vectors generally point? Explain this result in terms of the force acting on the satellite.

2. In which part of the orbit do your results differ? Why?
3. How does the force on the satellite depend on its distance from the Earth?

Method 3

This method involves use of the PSSC film "Elliptical Orbits" (available through PEMC in Richmond). Follow the instructions given during the course of the film.

Method 4

This method involves the use of Microcomputer Simulation. If a microcomputer is available to you, try programming the motion of a satellite of a certain mass moving around an object of much greater mass under the influence of gravitational force. Display the orbit of the satellite under these circumstances and determine its properties.

All Methods

Regardless of the steps you have followed up to this point, complete the investigation as follows.

Additional Activities

1. Find out the meaning of the word "eccentricity" when applied to ellipses. Use a reference book to find out the eccentricity of the orbits of the planet Pluto and of Halley's Comet. Choose a suitable scale and draw both orbits using the sun as focus.
2. Bodies that are moving too rapidly around the sun move in parabolic or hyperbolic paths instead of elliptical orbits. A general equation for all conics, using polar co-ordinates, is

   $R = \dfrac{ED}{1 - E\cos\theta}$ where E is the eccentricity and D is

   the distance from the focus to the directrix. The focus is at (0,0). Graph the three conic curves on the same graph using $D = 10$ and $E = 0.8$ (ellipse), $E = 1.0$ (parabola), and $E = 1.2$ (hyperbola). Use $\theta = 180°, 160°, 140°$, etc. A protractor and ruler will be necessary to measure θ and R unless you use polar graph paper.

Practice Problems

1. If a tenth planet were discovered with a mean orbital radius of 1.20×10^{13} m, what would be its period of revolution around the Sun?

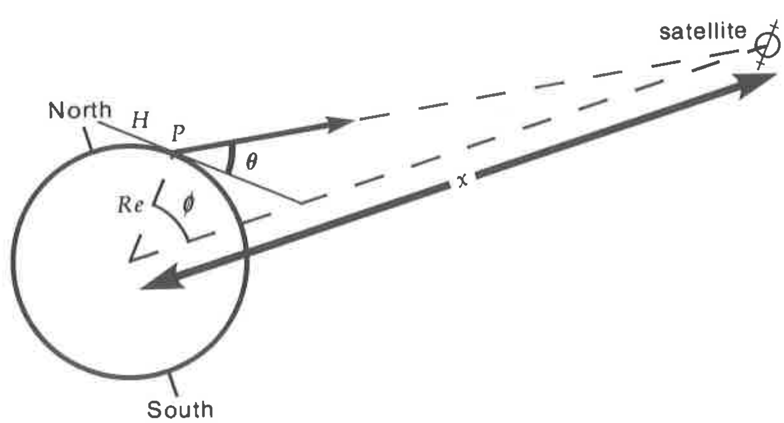
2. Halley's Comet moves around the Sun in a highly elliptical orbit. Its period of revolution is 76 a. What is the mean radius of its orbit?

3. Show that the constant k_E for satellites of the Earth is 3.02×10^{-6} times the corresponding constant k_S for satellites of the Sun.

4. Show that the acceleration due to gravity g on the surface of a planet is proportional to the product of the planet's density ρ and its radius R with a proportionality constant of $4/3\pi G$. Assume that the planet is spherical and that its density is uniform.

5. A synchronous television transmitting satellite is in orbit around the equator with a period of 24 h. At which angle θ above the southern horizon would you point a parabolic receiving antenna in order to obtain the strongest signal? (Use diagram #033 for additional information. Also see Appendix 9.)

#033 Diagram for Problem 5: Find x and θ; R_e = radius of the Earth (6.4×10^6 m), ϕ = latitude of location P (49° in Vancouver), and H is the horizon line at location P



False facts are highly injurious to the progress of science, for they often endure long; but false views, if supported by some evidence, do little harm, for everyone takes a salutory pleasure in proving their falseness.

Charles Darwin

Electrostatics

Investigation 7:

Coulomb's Law

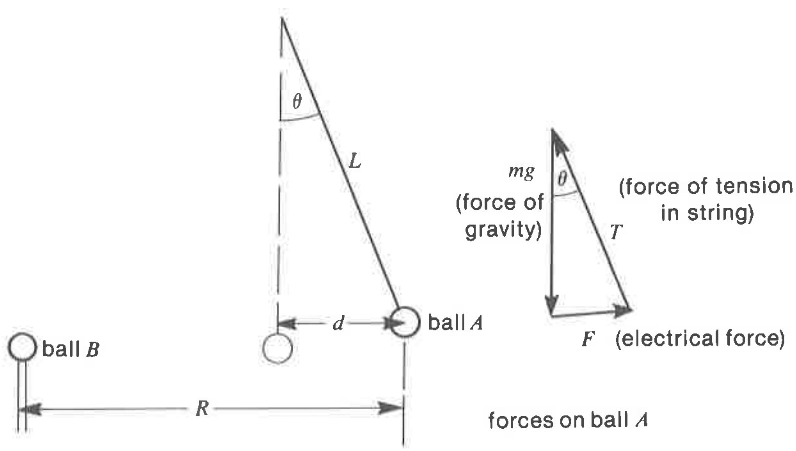
The electrical force that electrically charged particles can exert on each other is much stronger than the gravitational force. The strength of the electrical force can be expected to depend on the magnitude of the charges and on the distance between them. The formula governing the exact nature of the relationship for very small charged particles has become known as Coulomb's Law (after Charles-Augustin Coulomb, 1736–1806).

The methods used to study Coulomb's Law in this investigation all involve balancing the electrical force with other forces that are easier to measure.

Method 1

This method involves the use of a PSSC-type Coulomb's Law apparatus such as that shown in figure #035. A pith ball (ball *A*) is suspended in such a way that its movement is confined to one plane, and is electrically charged by means of an acetate strip. A second pith ball (ball *B*) is charged similarly and brought close to ball *A*. When this occurs, ball *A* is repelled. The forces acting on it are shown in figure #034.

#034 Forces involved in a study of Coulomb's law

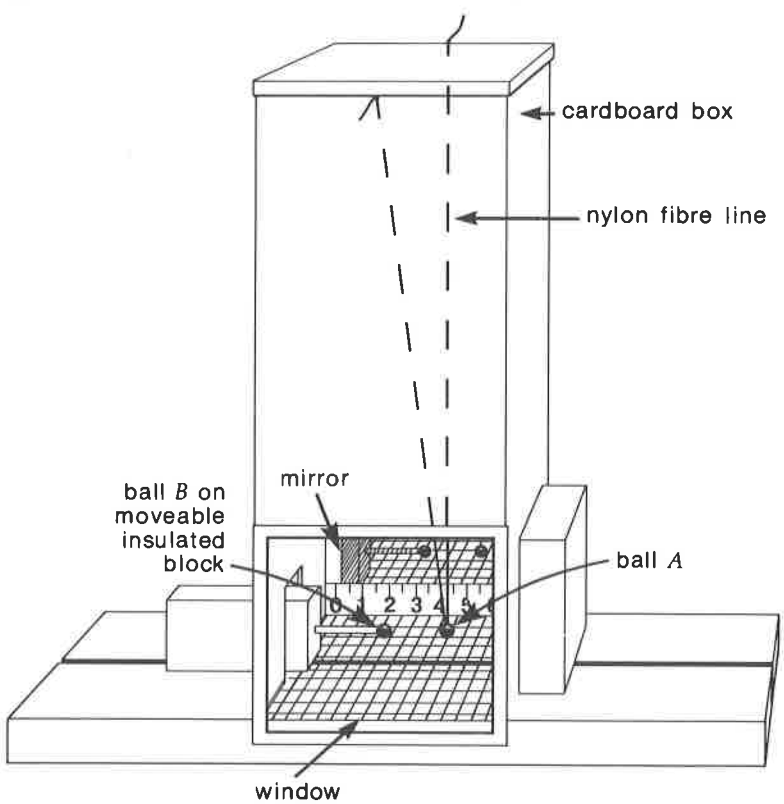


From the first diagram, $d/L = \sin\theta$. (See Appendix 9.) If θ is kept small, then, from the second diagram, $\vec{F}$ is almost perpendicular to $\vec{T}$, so $\frac{F}{mg} = \sin\theta$. Therefore $\frac{F}{mg} = \frac{d}{L}$, or $F = \frac{mg}{L}d$. Since g and L are constant, F is directly proportional to d, so to investigate the relationship between F and R you need only investigate the relationship between d and R, which are both relatively easy to measure.

Apparatus

PSSC-type Coulomb's Law apparatus (see figure #035)
acetate strip and cloth square

#035 Coulomb's Law apparatus



Procedure

1. Suspend one pith ball, *A*, from the top of the box by the nylon fibre as shown. (The double suspension is needed to restrict the movement of the ball to a plane.) Make sure that ball *A* and ball *B* are at the same height when they are uncharged.
2. Charge both balls using a charged acetate strip. Record the position of ball *A* when ball *B* is still a long way outside the protective box.
3. Slide ball *B* on its insulated block towards ball *A* until *A* is deflected as much as possible. Observe how the deflection changes with time as the charges on both balls leak away to the surrounding air.
4. Recharge the balls, if necessary, and again bring ball *B* towards ball *A*. This time record the positions of both balls for various positions of *B*. (You will probably have to work fairly quickly, depending on how rapidly the charges disappear.)

Method 2

This method is similar to Method 1, but involves the use of slightly different apparatus. Start by reading the discussion at the beginning of Method 1 and examining figures #034 and #035.

Apparatus

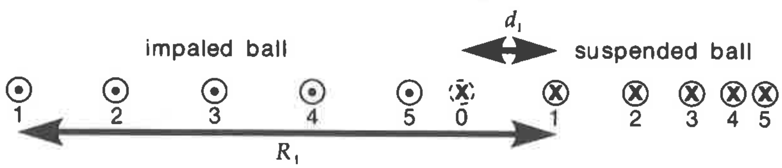
long string
2 pith balls
acetate strip and cloth square
overhead projector
insulating stick

Procedure

1. Suspend a pith ball directly over the centre of an overhead projector's writing surface, using a long string rigged in the V suspension system shown in figure #035. Impale a second pith ball on a sharpened popsicle stick (or another insulating handle).
2. Adjust the height of the suspended ball so that the centres of the two balls are at the same level.
3. Focus the overhead projector so that a reasonably clear image is shown on a large piece of paper taped to a nearby wall. Mark on the paper the equilibrium position of the suspended ball.

4. Charge the two balls with a charged acetate strip. Then slide the impaled ball slowly towards the suspended ball, making sure that the ball does not touch the surface of the overhead projector. On the paper, mark the positions of both balls for various positions (at least 6) of the impaled ball (as in figure #036).

#036 Relative positions of two charged balls (corresponding positions have the same numbers)



Method 3

If the pith balls discharge too rapidly to complete the experimental procedure of Method 1·easily, it is possible to use a method in which ball *A* is replaced by a balloon and ball *B* by a Van de Graaff generator.

However, if you use balloons, you should keep them far enough apart that they can be considered "point" objects compared to the distance between them. In practice, this will probably be impossible, and you should be prepared for deviations from Coulomb's Law for point changes.

Start by reading the discussion at the beginning of Method 1 and examining figures #034 and #035.

Apparatus

Van de Graaff generator
balloon about the same size as the Van de Graaff generator
long string

Procedure

1. Suspend the balloon from a V-shaped string (as in figure #035) as long as possible.
2. Make sure that the base of your Van de Graaff generator is grounded. Set it near the balloon and adjust the balloon so that it is at the same height as the dome of the generator.
3. Set up a distant light so that it casts a shadow of the balloon and the generator on to a large piece of paper taped to a nearby wall. Mark on the paper the equilibrium position of the balloon.

4. Turn on the generator and push the balloon into contact with it. On the paper, mark the positions of the centres of the dome and the balloon for several different positions of the dome. To move the generator, use a long wooden stick.

All Methods

Regardless of the apparatus and procedure used, complete the investigation as follows.

Data Analysis

1. Measure (Methods 2 and 3) or calculate (Method 1) the value of R and the corresponding values of d.
2. Graph d versus R (see Appendix 3) and determine the mathematical relationship between them. (See Appendix 4.)

Discussion of Results

1. How does the force exerted by one charged object on another depend on the distance between them? (See Appendix 7.)
2. For each of the difficulties you encountered while doing this investigation, suggest a method which might overcome or reduce the problem.

Additional Activities

1. If you are using pith balls, use your apparatus to determine the relationship between the electrical force and the quantity of charge on each pith ball. You can vary the amount of charge on each ball by distributing an original charge between 2, 3, or 4 pith balls of equal size.
2. Suspend three pith balls by long strings so that they are at the same height and in contact with one another. Charge the balls equally and observe the positions of the balls after charging. Repeat this step using 4 and 5 balls. Can you predict theoretically where and how far apart they come to equilibrium?

Practice Problems

1. (a) If the distance between two charged particles is doubled, by what factor is the electrical force between them multiplied?

   (b) If the magnitude of the charge on each particle is increased by a factor of 3, by what factor is the force between them multiplied?

   (c) If both of the above changes are made, by what factor will the original force be multiplied?

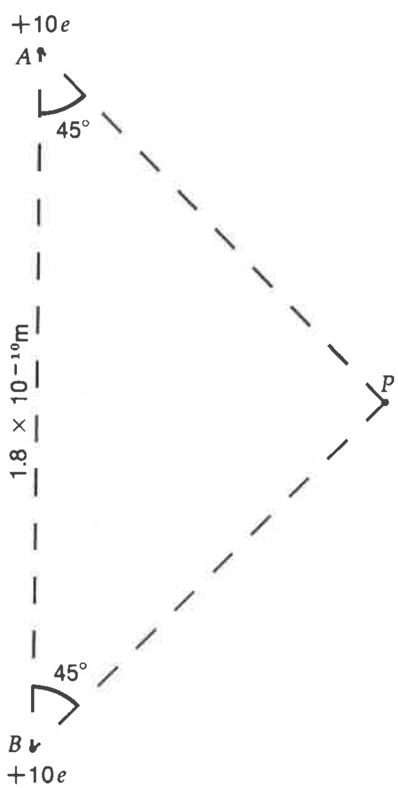
2. Two charged objects, 0.30 m apart, exert a force of 2.0 N on each other. The charge on one object is 5.0×10^{-6} C. What is the magnitude of the other charge?

3. (a) What is the magnitude of the electrical force between a proton, with a charge of $+1.6 \times 10^{-19}$ C, and an electron, with a charge of -1.6×10^{-19} C, separated by a distance of 3.0×10^{-11} m?

   (b) What is the magnitude of the gravitational force between a proton, of mass 1.7×10^{-27} kg, and an electron, of mass 9.1×10^{-31} kg, separated by a distance of 3.0×10^{-11} m?

   (c) What is the ratio of the magnitude of the electrical force to the magnitude of the gravitational force for a proton and an electron?

#037 Diagram for Problem 4:
$e = 1.6 \times 10^{-19}$ C



4. (a) Using figure #037, calculate the electrical force on an electron at point *P* due to the charges at *A* and *B*.

   (b) Calculate the electrical force on an electron at point *P* if the charge at *B* is changed to $-10\ e$.

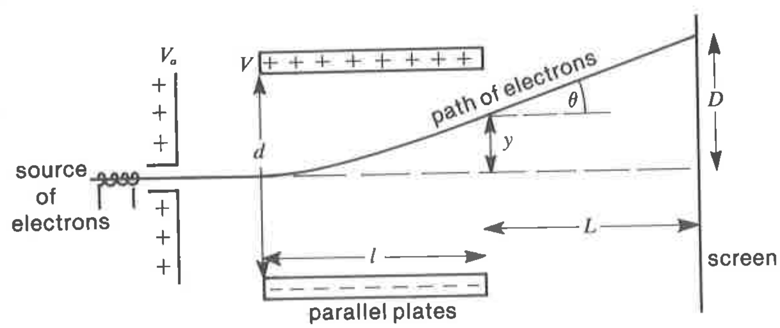
Science is nothing but trained and organized common sense.
Thomas H. Huxley

Investigation 8:

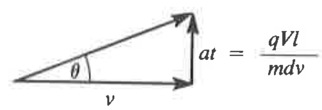
Deflection of an Electron Beam by an Electric Field

Television picture tubes are based on the operation of a cathode ray tube, although there are differences. The cathode ray tube produces a narrow beam of electrons that is directed towards a distant screen. When the beam strikes the screen, the screen fluoresces, producing a bright spot of light. On its way towards the screen, the electron beam passes between parallel plates that can be electrically charged so that they deflect the beam. With two sets of plates, one in the vertical plane and one in the horizontal plane, and with various potential differences between the plates, the beam can be deflected to any point on the screen. The much simplified figure #038 shows the key elements of a cathode ray tube.

#038 Key elements of a cathode ray tube



An electron, of mass m and charge q with a speed of v enters the region between the plates along the centre line. If the plates are charged to a potential difference of V, there is an electric field of strength E between them and the electron experiences an electric force F of magnitude $F = qE$ or $F = \frac{qV}{d}$. This force causes the electron to accelerate perpendicular to its original direction with an acceleration of $a = \frac{F}{m}$ or $a = \frac{qV}{md}$. The electron continues to accelerate during the time t it spends between the plates. ($t = \frac{l}{v}$ where l is the length of the plates.) By the time the electron leaves the region between the plates, it has been deflected a distance of $y = \frac{1}{2} at^2$ or $y = \frac{1}{2} \frac{qVl^2}{mdv^2}$. After it leaves the plates, the electron is no longer accelerated; it simply continues toward the screen in a straight line. The angle between its new path and its original path is θ and depends on how fast the electron is now travelling perpendicular to the plates. This perpendicular velocity is $at = \frac{qVl}{mdv}$. Then, from the velocity triangle shown,



$$\tan\theta = \frac{qVl}{mdv^2}$$

(See Appendix 9.) It can be seen that

$$\begin{aligned} D &= y + L\tan\theta \\ &= \frac{qVl^2}{2\,mdv^2} + L\,\frac{qVl}{mdv^2} \\ &= \frac{qVl(l/2 + L)}{mdv^2} \end{aligned}$$

All of the values V, d, l, and L are easily obtained. The mass and the charge of an electron are also known. This leaves only v to be determined. If the electrons are produced by thermionic emission from the cathode and accelerated through a potential difference of V_a between the cathode and the anode, then they will enter the region between the plates with kinetic energy equal to
$\frac{1}{2} mv^2 = qV_a$.

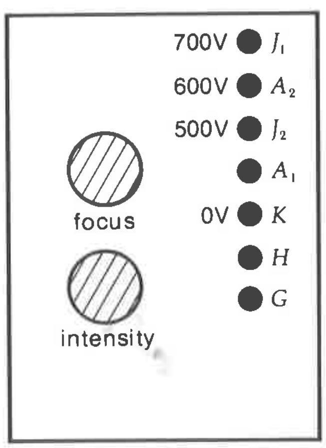
Therefore $v^2 = \frac{2qV_a}{m}$ and $D = \frac{1}{2} \frac{l(l/2 + L)V}{dV_a}$.

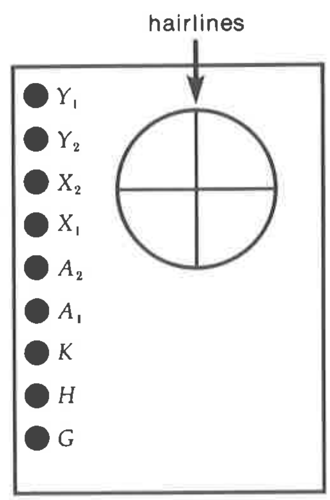
According to this theoretical equation, the visible deflection D is directly proportional to the deflecting voltage V and inversely proportional to the accelerating voltage V_a. In this investigation you will check this relationship for a beam of electrons in a cathode ray tube.

Apparatus

cathode ray tube
batteries (15 V transistor type)
potentiometer (10 MΩ)
voltmeter

#039 A standard cathode ray tube apparatus (top view). The terminals are usually color coded for ease of wiring. However, as models differ considerably, always check with the manufacturer's specifications before making any connection. The equipment is much easier to use if
(a) there is a 4.7 MΩ resistor between X_1 and X_2,
(b) there is a 4.7 MΩ resistor between Y_1 and Y_2,
(c) there is a solid wire connection between X_2 and Y_2, and
(d) there is a solid wire connection between X_2 and A_2.
The resistors and connections should be wired internally beforehand. However, do not make the connection in (a) if the cathode ray tube is to be used as a vacuum tube voltmeter in Investigation 14.





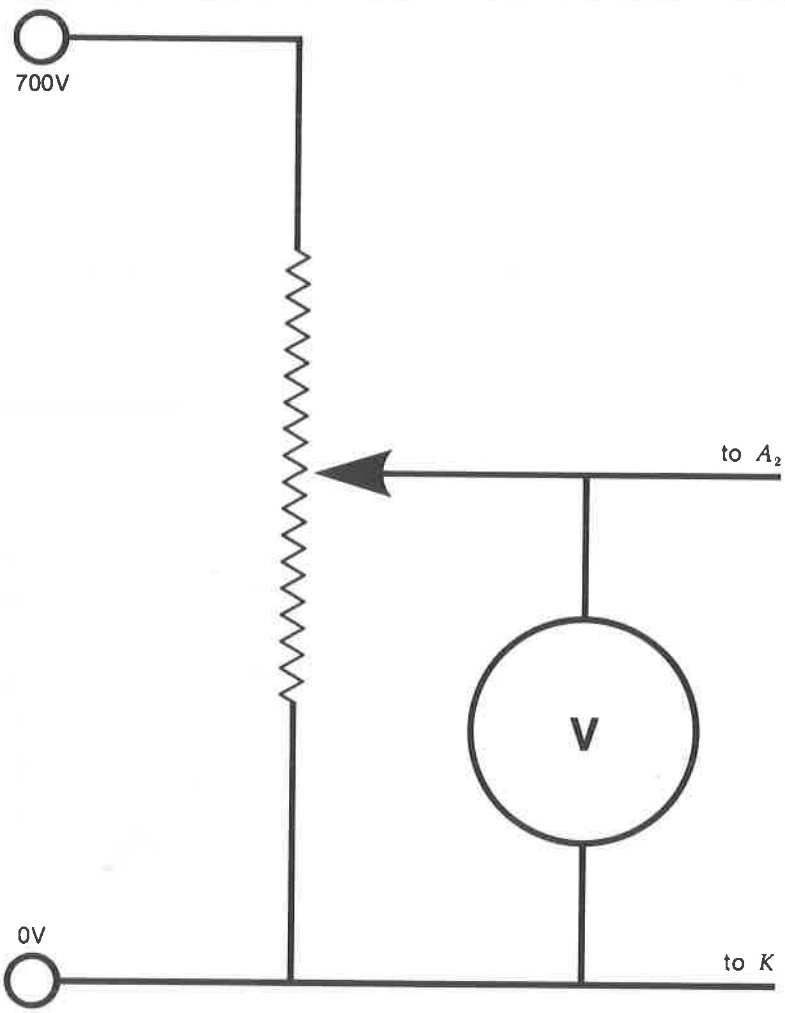
Procedure

1. Connect the cathode ray tube to its power supply, following the manufacturer's instructions. Use an accelerating potential of 700 V. Adjust the focus and intensity controls until a sharp but not too bright spot of light is obtained at the centre of the screen. You can tape hairs to the screen to mark the centre position.

2. Connect a 15 V battery across the X_1 and X_2 terminals. Measure the deflection of the spot of light. Repeat this operation with 30 V, 45 V, 60 V and 75 V batteries. Reverse the connections and observe the effect on the deflection.

3. Place a potentiometer and a voltmeter between the 700 V terminal post and K (figure #040).

#040 Placement of potentiometer and voltmeter for step 3



4. Connect a 30 V deflecting voltage across X_1 and X_2.
5. Set the accelerating voltage to the middle of its range and adjust the focus and intensity controls to obtain the sharpest spot of light possible.
6. Measure and record the deflection produced for at least 5 different values of the accelerating voltage.

Data Analysis

1. By graphing your data, D vs. V (see Appendix 3), find the relationship between the deflection and the deflecting voltage. (See Appendix 4.)
2. By graphing your data, D vs. $1/V_a$ (see Appendix 3), find the relationship between the deflection and the accelerating voltage. (See Appendix 4.)

Data Interpretation

1. If your tube has listed values of d, l, and L, compare the actual equations of your graphs with the theoretical equation. (See Appendix 6.)
2. How well do your experimental results compare with your theory?
3. Explain how the deflection depends on the polarities of the deflecting plates.

Additional Activities

1. Apply deflecting voltages to both the horizontal and the vertical deflection plates at the same time. Observe the resultant effect on the beam. Then pick a point somewhere on the screen and try to deflect the spot to that location by adjusting the two deflecting voltages. (Use potentiometers.)
2. Connect an alternating voltage to the horizontal plates. Explain the effect produced as the alternating voltage is increased. Connect the alternating voltage to both sets of plates at the same time. Try to explain the pattern produced. (Remember that an alternating voltage varies with time as a sine wave.)
3. If an audio frequency generator is available, try connecting it to one set of plates. Connect a 60 Hz alternating voltage to the other set of plates. (This activity is best done using an oscilloscope set on its internal sweep frequency [60 Hz] with the audio generator attached to the vertical deflection posts.) Adjust the magnitude of each applied voltage separately so that the maximum deflection produced is about one half of the screen size.

   Turn the generator to 60 Hz and apply both voltages at the same time. Try to obtain a circular pattern on the screen. You will probably have to make slight adjustments to both the frequency and the gain of the generator. Slowly increase

the frequency of the generator and observe the changes produced in the pattern on the screen. Predict the pattern when the frequency reaches 120 Hz, then test your prediction. Do the same for 180 Hz and 240 Hz.

Practice Problems

1. Electrons are accelerated from rest through a potential of 500 V in an electron "gun." Assuming that all of the work done on the electrons produces kinetic energy, what is the speed of the electrons leaving the gun ($m_e = 9.1 \times 10^{-31}$ kg, $q_e = 1.6 \times 10^{-19}$ C)?
2. Electrons which have been accelerated through a potential of 500 V pass between two parallel plates that are 6.0 cm long and 2.0 cm apart, with a potential of 150 V between them.
   (a) What is the energy of the electrons leaving the gun?
   (b) What is the velocity of the electrons down the tube if all of their energy is in the form of kinetic energy?
   (c) What is the electric field strength between the plates?
   (d) What is the tranverse force on the electrons between the plates?
   (e) What is the transverse acceleration of the electrons?
   (f) What is the time required for the electrons to pass through the region between the deflection plates?
   (g) What is the deflection of the electrons while passing through the region between the plates?
   (h) What is the transverse velocity of the electrons after leaving the region between the plates?
   (i) The screen is 20 cm beyond the end of the charged plates. What time is required for the electrons to travel this distance?
   (j) What is the size of the deflection as seen on the screen?
3. Do Question 2 for protons instead of electons ($m_p = 1.7 \times 10^{-27}$ kg, $q_p = 1.6 \times 10^{-19}$ C).

The universe is not to be narrowed down to the limits of our understanding, but our understanding must be stretched and enlarged to take in the image of the universe as it is discovered.

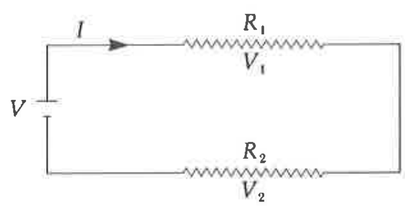
Sir Francis Bacon

Circuitry

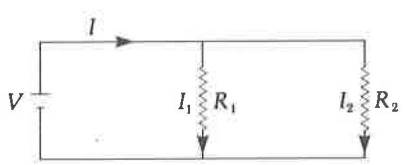
Investigation 9:

Series and Parallel Circuits

#041 A series connection: both resistors have the same current through them. V_1 is the potential drop across the resistor R_1 and V_2 is the potential drop across the resistor R_2.



#042 A parallel connection: both resistors have the same potential difference across them. I_1 and I_2 are the currents through the two branches of the circuit.



Many electrical resistors obey Ohm's Law, $V = IR$, where V is the potential difference across the resistor (in volts), I is the current through the resistor (in amperes), and R is the resistance of the resistor (in ohms).

When two or more resistors are connected end to end so that the same current passes through each in turn, they are said to be connected in series (see figure #041). If the resistors are connected so that the current can take different branches as it travels through the circuit but the potential difference across each resistor is the same, they are said to be connected in parallel (see figure #042).

Electric circuits also obey Kirchhoff's Laws, which are restatements of the Laws of Conservation of Charge and Conservation of Energy.

First Law: At any junction in a circuit, the sum of the currents entering the junction is equal to the sum of the currents leaving the junction.

Second Law: The sum of the changes in electric potential around any branch of a circuit is zero.

The equivalent resistance, R, of a number of resistors in series or in parallel is the single resistance that could replace all of the separate resistances in the circuit without changing the current through the power source.

For the circuit shown in figure #041, it is possible to predict the equivalent resistance of the two resistors as follows:

According to Kirchhoff's Second Law,

$$V = V_1 + V_2 .$$

Therefore, according to Ohm's Law,

$$IR = IR_1 + IR_2$$

and $R = R_1 + R_2$.

For the circuit shown in figure #042, the equivalent resistance of the two resistors can be predicted as follows.

According to Kirchhoff's First Law,

$$I = I_1 + I_2 .$$

Therefore

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

and

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} .$$

In this investigation, you will check these equations, and, in so doing, check Kirchhoff's Laws.

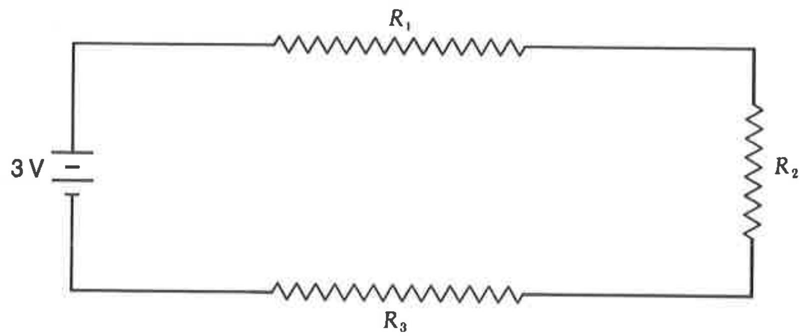
Apparatus

voltmeter
ammeter
3 resistors (between 10 Ω and 100 Ω and not all equal)
2 1.5 V batteries
connecting wire

Procedure

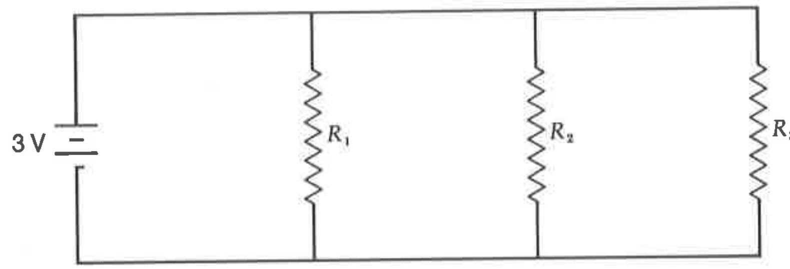
1. Set up the circuit shown in figure #043. Use the voltmeter to measure the potential difference across the battery and across each of the three resistors. Record these values.

#043 Series circuit diagram (for step 1)



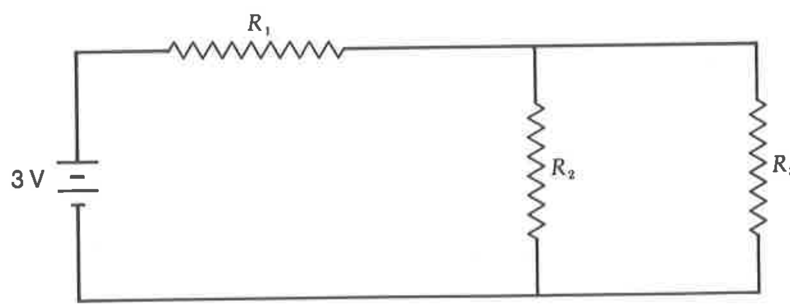
2. Use the ammeter to find the current through the battery and through each resistor.
3. Set up the circuit shown in figure #044. Measure the potential difference across the battery and across each of the resistors.

#044 Parallel circuit diagram (for step 3)



4. Measure the current through each branch of the circuit.
5. Set up the circuit shown in figure #045. Measure the potential difference across the battery and across each of the resistors.

#045 Series/parallel circuit diagram (for step 5)



6. Measure the current through the battery and through each resistor.

Data Analysis

1. For each of the circuits, calculate the equivalent resistance of the circuit from the measured potential difference V across the battery and the current I through the battery. Remember that $R = \frac{V}{I}$.
2. For each of the circuits calculate the equivalent resistance of the resistors, using the theory given in the introduction.

Discussion of Results

1. Calculate the percentage difference between the actual equivalent resistance and the theoretical equivalent resistance. (See Appendix 5.)

2. Are Kirchhoff's rules satisfied, within experimental error, in each of your circuits (i.e., does $\Sigma V = 0$, and does $\Sigma I_{in} = \Sigma I_{out}$)? (Two sources of error are the resistance of your ammeter and the lack of resistance in your voltmeter.)

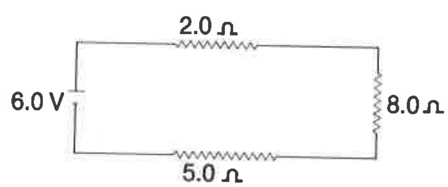
Additional Activities

1. Analyse a series or parallel circuit of your own design. Check your predictions by experiment.
2. Construct a parallel circuit that has a battery in more than one branch of the circuit. Use Kirchhoff's rules to predict the current through and the potential difference across each resistor in the circuit. Check your predictions by experiment.

Practice Problems

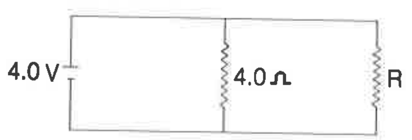
1. Calculate the equivalent resistance of the resistors in the circuit shown in figure #046.

#046 Circuit diagram for Problem 1



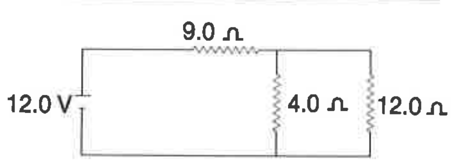
2. In figure #047, what is the value R of the unknown resistance if a current of 4.0 A leaves the power supply?

#047 Circuit diagram for Problem 2

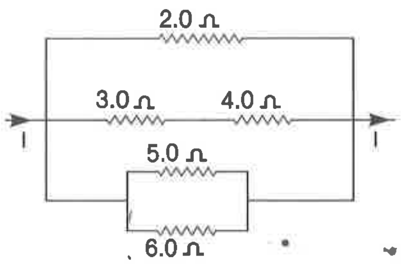


3. For figure #048, calculate the current through the 12 Ω resistor.

#048 Circuit diagram for Problem 3



#049 Circuit diagram for Problem 4



4. For figure #049, find the equivalent resistance of the resistors shown.

5. A flashlight battery has a voltmeter connected across its terminals. The voltmeter reads 1.50 V. (Assume that the voltmeter is perfect, with an infinite resistance, drawing no current.) The voltmeter is removed and the battery is connected to a small light bulb which has a resistance of 2.0 Ω. An ammeter, of negligible resistance, placed in the circuit measures 0.71 A.

   (a) What is the potential drop across the bulb?

   (b) What is the internal resistance of the battery?

   (c) If the voltmeter were reconnected across the terminals of the battery with the light bulb still in the circuit, what voltage would it record?

Order must be discovered and, in a deep sense, it must be created.

J. Bronowski

Electromagnetism

Investigation 10:

Deflection of an Electron Beam By a Magnetic Field

For many years electricity and magnetism were considered to be two separate phenomena. It was not until 1820, when Hans Christian Oersted discovered during the course of a lecture that an electric current produced a magnetic field, that the connection between the two was realized. Eleven years later, Michael Faraday demonstrated that a changing magnetic field produces an electric current. The new subject was named electromagnetism.

In Investigation 8 you studied the effect of an electric field on a beam of electrons. The same apparatus can be used to show the effect of a magnetic field on the beam. This effect can be easily observed by bringing a reasonably strong bar magnet close to the glass of a Crookes tube (which produces a beam of electrons when connected to an induction coil). However, for this investigation you will use the magnetic field produced by a current in a solenoid, a long coil of wire.

You will begin by finding the relationship between the deflection of the beam in a cathode ray tube and the current through the solenoid (which controls the magnetic field strength). Then you will find the relationship between the deflection of the beam and the speed of the electrons in the beam.

In Investigation 8, you saw that the deflection y of the beam was given by

$$
\begin{aligned}
y &= \frac{1}{2}at^2 \\
&= \frac{1}{2}\frac{F}{m}\left(\frac{l}{v}\right)^2 \\
&= \frac{1}{2}\frac{Fl^2}{mv^2}
\end{aligned}
$$

where F is the magnitude of the deflecting force, l is the length of the region where the deflecting force acts, m is the mass of an electron, and v is the speed with which the electron passes through the deflecting region. However, the derivation of this expression depends on the fact that the force has a constant direction. (The

electric force is always perpendicular to the deflecting plates.) But the magnetic force on a beam of electrons does not have a constant direction; it is always perpendicular to the direction of the electrons, so its direction changes as the electrons are deflected.

However, if the deflection of the electrons is kept small, the direction of the electrons does not change much and so the direction of the magnetic force does not change much. In that case, the deflection of the electrons will be approximately proportional to the magnetic force, just as it was proportional to the electric force in Investigation 8.

In Investigation 8, y (the deflection of the electrons as they left the region between the plates) was different from D (the deflection actually observed on the screen) because the deflecting plates were some distance away from the screen. In this investigation, you can place the solenoids so that the electrons hit the screen as soon as they have passed through the region between the solenoids, so D is the same as y.

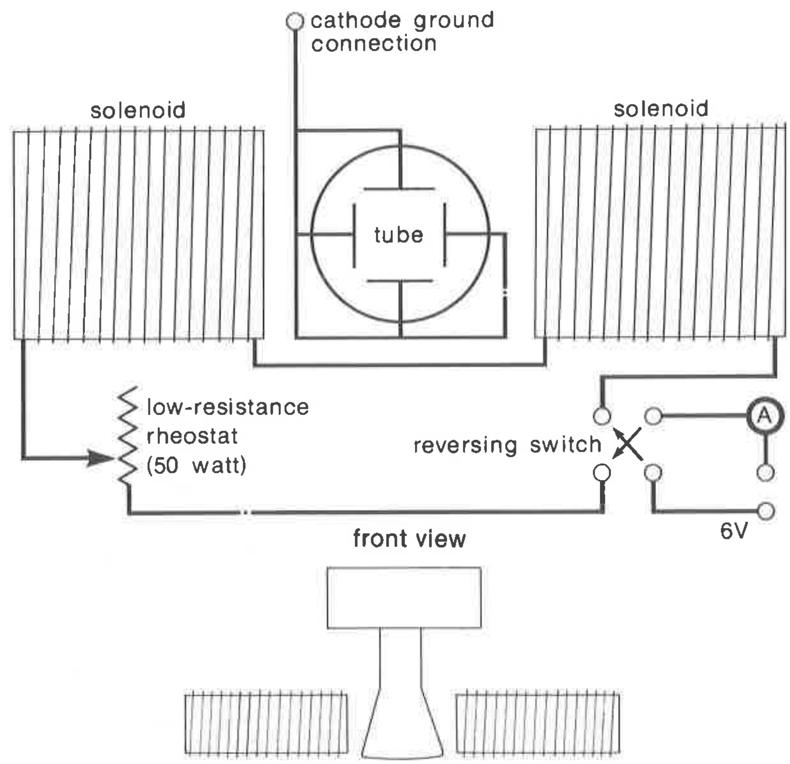
Apparatus

cathode ray tube
2 solenoids
6 V power supply
rheostat (50 W)
potentiometer (10 MΩ)
voltmeter
ammeter

Procedure

1. Place the cathode ray tube on its side so that the end of the tube is close to the surface of a table. Place the solenoids as close as possible to the tube with one on each side of the tube. Connect the cathode ray tube as you did in Investigation 8 and the solenoids as shown in figure #050.

 # 050 Apparatus for using a magnetic field to deflect an electron beam



Be sure to connect the solenoids so that the current through each has the **same** direction (clockwise or counter-clockwise). Connect the X_1, X_2, Y_1, and Y_2 plates to A_2 to avoid any electric deflection.

2. Set the accelerating voltage of the beam at 700 V. Adjust the focus and intensity controls to produce a sharp, bright spot at the centre of the screen.

3. Turn on the power supply to the solenoids. Observe the deflection produced as the current is varied (do not exceed 5 A in the solenoids). With the current set at the middle of its range, refocus the spot.

4. Measure the deflection D of the electron beam for at least 5 different values of the current I, but do not let the deflection exceed about 2 cm. (If the electrons are deflected more than this, you cannot expect your data to agree with your theory.)

If the spot "smears out" into a line, use the value of the **maximum** deflection (the extreme end of the line). Also reverse the current to find the deflection in the opposite direction, and average your results.

5. Put a potentiometer and a voltmeter between the 700 V output of the cathode ray tube power supply and K (as in step 3 of the procedure for Investigation 8).

6. Set the current through the solenoids at 4.0 A. Measure the deflection produced for at least 8 different values of the accelerating voltage V_a, but do not let the deflection exceed about 2 cm. Use the maximum intensity of the beam.

Data Analysis

1. Graph D versus I (see Appendix 3) and find the relationship between them. (See Appendix 4.)

2. Graph D versus V_a (see Appendix 3) and find the relationship between them. (See Appendix 4.)

Interpretation of Results

1. From Investigation 8, you know that

$$D = \frac{1}{2}\frac{Fl^2}{mv^2}.$$

l and m are constant, and, in the first part of the investigation, v was constant because you did not change the accelerating voltage. Hence

$$D \propto F.$$

From your graph,

$$D \propto I$$

so $\quad F \propto I.$

State this relationship in words, giving the meaning of each variable.

2. In the second part of the investigation, you varied the accelerating voltage V_a, so the speed v of the electrons was not constant. Therefore

$$D \propto \frac{F}{v^2}.$$

Now, according to your graph,

$$D \propto \frac{1}{\sqrt{V_a}}.$$

You know that as the electrons were accelerated through the potential difference V_a, they gained kinetic energy

$$\frac{1}{2}mv^2 = qV_a$$

where q is the charge on an electron. Therefore

$$V_a \propto v^2$$

and $\sqrt{V_a} \propto v$

so $D \propto \frac{1}{v}$.

If $D \propto \frac{F}{v^2}$

and $D \propto \frac{1}{v}$

then $\frac{F}{v^2} \propto \frac{1}{v}$

so $F \propto v$.

State this relationship in words, giving the meaning of each variable.

2. How could you distinguish between a deflection due to a magnetic field and a deflection due to an electric field?
3. What sources of error are inherent in this experiment?

Additional Activities

1. Plug a long-filament clear light bulb into a 110 V ac circuit. Bring a strong bar magnet close to the glass and observe the effect produced. Try to explain why this effect occurs.

Practice Problems

1. An electron moves at a speed of 1.5×10^6 m/s at right angles to a magnetic field of 3.0×10^{-2} T.
   (a) What force acts on the electron (magnitude and direction)?
   (b) What centripetal acceleration does the electron have?
   (c) What would the acceleration of a helium nucleus ($q = +3.2 \times 10^{-19}$ C and $m = 6.7 \times 10^{-27}$ kg) be in the same circumstances?
2. A deflection of 1.8 cm is observed on a cathode ray tube when there is a current of 2.5 A through the solenoids. What would the deflection be for a current of 4.2 A?

3. A deflection of 2.5 cm is observed on a cathode ray tube when the accelerating voltage is 600 V. What voltage would cause a deflection of 4.0 cm if the current through the solenoids were constant?

4. (a) What is the relativistic mass of an electron travelling at 2.0×10^8 m/s?

   (b) How does the deflection of an electron by a magnetic field change if relativistic effects are taken into account (i.e., would the relativistic deflection be greater or smaller than the non-relativistic deflection)? Explain your reasoning.

Science is not a sacred cow. It is a horse. Don't worship it. Feed it.

Aubrey Eben

Investigation 11:

The Current Balance

In this investigation you will study how a magnetic field affects a wire carrying an electric current I_s (see figure #051). The magnetic field is produced by another current I_B through a long coil of wire (a solenoid). You will also examine how the strength B of the solenoid's magnetic field depends on the size of the current I_B through the solenoid.

When a charged particle such as an electron moves through a magnetic field (unless it is travelling parallel to the field), it experiences a force F. In Investigtion 10, you saw that this force was proportional to the speed v of the particle. For this reason, the magnetic field strength B is defined to be the magnetic force **per coulomb** of charge q **per metre/second** of speed v possessed by the particle (if the particle is travelling at right angles to the field). In that case, we write

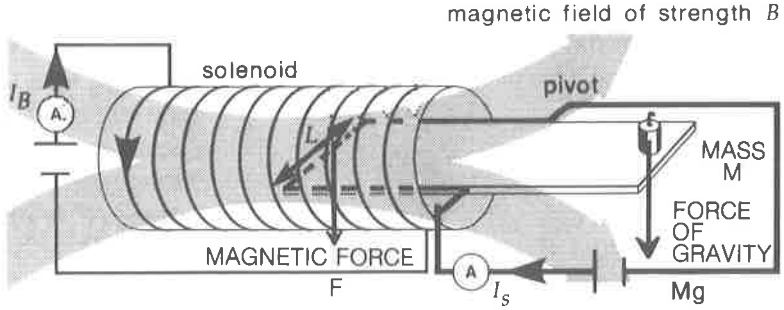
$$B = \frac{F}{qv}$$

or $$F = Bqv\,.$$

This force is perpendicular to the directions of both the magnetic field and the velocity of the particle.

If the particle is one of a steady stream of particles moving through a conductor of length L, then $qv = IL$. If the conductor is set at right angles to the field, then $B = F/IL$ or $F = ILB$. This magnetic force on a conductor can be measured by balancing it with the force of gravity on a small mass in a device called a current balance (see figure #051).

#051 Current balance apparatus



Only the length of the current-carrying strip marked L has a force acting on it, as this is the only section of the strip that is not parallel to the magnetic field of the solenoid. The direction of the current I_s through this part of the strip can be arranged so that the force $\vec{F}$ acting on it is downward (as you can predict using the right-hand rule) and its magnitude F can be measured by counter-balancing it with the force of gravity Mg on the mass.

The magnetic field of a solenoid is uniform in magnitude and direction within the interior of the solenoid, if the solenoid is considerably longer than its diameter.

In this investigation, you will first measure the magnetic field inside the solenoid for a given current through the solenoid (by measuring the force F per ampere of current I_s through the strip per metre of length L of the strip). Then you will check the relationship you found in Investigation 10: that the magnetic force on the strip and hence the magnetic field inside the solenoid is directly proportional to the current I_B through the solenoid.

Apparatus

PSSC-type current balance apparatus
milligram masses
2 rheostats (50 W)
2 ammeters
power supply of 6 V (10 A maximum) or, if available, two power supplies, since, if one is used, varying either current is likely to vary the other also.

Procedure

1. Connect the power supply to the solenoid through a rheostat and an ammeter so that the current can be controlled and measured.

2. Connect another power supply (if you have one; if not, use the same one) to the strip through a rheostat and an ammeter. Make sure that the contact points of the strip are clean.

3. Place a 10 mg mass (M) on the end of the current balance strip. Turn on the power supply with the rheostats set so that the currents are minimal. Increase the current through the solenoid until it measures 4.0 A. Then keep this current constant. (Be sure that the solenoid does not overheat.) Gradually increase the current through the strip until it balances. Record the current through the solenoid (I_B) and the current through the strip (I_s) as well as the force on the end of the balance ($F = Mg$).

4. Repeat the procedure using heavier masses on the balance strip until the maximum possible strip current is reached. Keep checking the solenoid and both rheostats for overheating. Then turn off the power supplies and remove the last mass.
5. Place a 10 mg mass on the balance strip and turn on the power supply. Adjust the current through the strip until it is 2.0 A (to avoid melting the contact points). Then keep this current constant.
6. Gradually increase the current through the solenoid until the strip balances. Record I_B, I_s, and the force on the strip.
7. Add progressively heavier masses to the balance strip until equilibrium can no longer be achieved. Keep checking the rheostats and the solenoid for overheating.

Data Analysis

1. Plot F versus I_s (see Appendix 3) and find the relationship between them (see Appendix 4).

   If this graph goes through the origin as we expect, its slope is

   $$\text{slope} = \frac{F}{I_s}$$

   The magnetic field strength B inside the solenoid is

   $$B = \frac{F}{I_s L} = \frac{\text{slope}}{L}$$

   Calculate the magnitude of the field inside your solenoid when the current I_B through the solenoid is 4.0 A.

2. Plot F versus I_B (see Appendix 3) and find the relationship between them (see Appendix 4).

   If this graph goes through the origin as we expect, its slope is

   $$\text{slope} = \frac{F}{I_B}$$

   The magnetic field strength B inside the solenoid *per ampere of current I_B through the solenoid* is

   $$k = \frac{B}{I_B} = \frac{F/(I_s L)}{I_B} = \frac{\text{slope}}{I_s L}$$

   Calculate the magnetic field strength per ampere inside your solenoid.

Discussion of Results

1. In determining the constant k for your solenoid, you have *calibrated* it. In future, you can calculate the magnetic field strength inside this particular solenoid for any current I_B through it as follows: $B = kI_B$. Label this solenoid with its calibration constant k, including the unit.

2. Theoretically, the field inside a solenoid which is long compared to its diameter is

$$B = \mu_0 \left(\frac{N}{l}\right) I_B$$

where N is the number of turns of wire on the solenoid in a length l of the solenoid and $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$ (called the magnetic permeability of free space). If possible, determine the N/l ratio for your solenoid and calculate the strength of the magnetic field per ampere of current through the solenoid, B/I_B, using this theoretical expression, and compare it with your measured value of k (see Appendix 5).

3. How does the field inside a solenoid vary with the current through the solenoid? (See Appendix 7.)

4. Describe any factors that could have led to differences between your experimental results and the predicted results.

Additional Activities

1. Design an experiment to show that the field produced by a solenoid is uniform within the interior of the solenoid. Also try to determine how quickly the strength of the magnetic field decreases with the distance away from the end of the solenoid.

2. Connect the strip and the solenoid in series so that the same current (I) runs through both. Then determine theoretically the relationship between F and I and check your results experimentally.

Practice Problems

1. (a) An air-cored solenoid has 500 turns/cm of wire wound on a cylinder. What is the strength of the magnetic field (B) produced near the centre of the solenoid if the current through the windings is 3.0 A?

   (b) What force would act on a wire placed along the central axis of the solenoid if the current through it was 4.0 A?

(c) What force would act on a wire of length 6.0 cm carrying a current of 1.5 A placed perpendicular to the field inside the solenoid?

2. The field strength inside a solenoid is 0.030 T. The length of a conducting strip at right angles to the magnetic field is 4.2 cm. How much current is there in the strip if it requires a 60 mg mass to balance the magnetic force on the strip?

3. In a current balance there is the same current, *I*, through both the solenoid and the strip. If a current of 1.5 A "balances" a mass of 0.10 g, what current would "balance" a mass of 0.30 g? (Find the relationship between *F* and *I* and use it to solve the problem.)

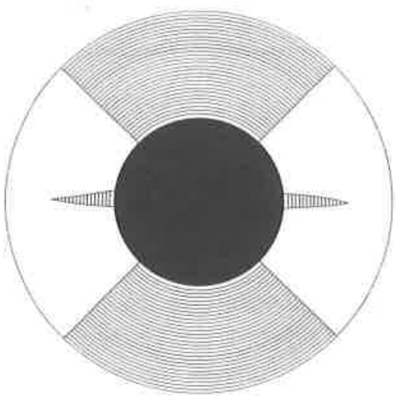
It is easy in experimentation to be deceived, and to think we have seen and discovered what we desire to see and discover.

Luigi Galvani

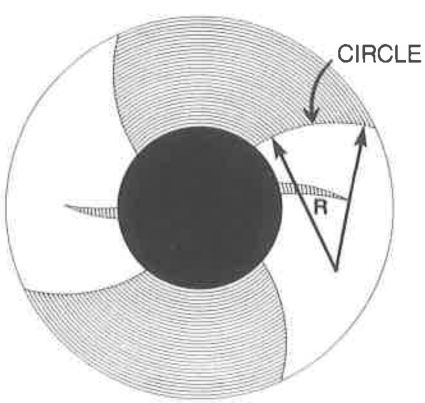
Investigation 12:

The Mass of an Electron

#052 Fan shaped beams of light created by electrons striking the disc (the anode) in a tuning eye tube



#053 Distortion of fan shaped beams of light when tuning eye tube is placed inside a solenoid



In a tuning eye tube, electrons are produced by a heated cathode at the centre of a shallow, funnel shaped, fluorescent anode. They are accelerated towards the anode by an accelerating voltage (V_a) between the cathode and the anode. The effect when the electrons strike the anode is of two fan-shaped beams of light with straight edges (see figure #052). When the tube is placed inside a solenoid, however, the electrons are deflected by the magnetic force into a circular arc of radius R (see figure #053).

The magnitude of the magnetic force is $F = qvB$ where q is the charge on an electron, v is its speed, and B is the strength of the magnetic field inside the solenoid. It is a centripetal force because it acts at right angles to the velocity of the electrons,

so $qvB = \dfrac{mv^2}{R}$ or $R = \dfrac{mv}{qB}$.

Now $qV_a = \dfrac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2V_aq}{m}}$ and $R = \dfrac{m}{qB}\sqrt{\dfrac{2V_aq}{m}}$.

By solving this equation for m, we obtain

$$m = \frac{R^2qB^2}{2V_a}.$$

From this relationship, we can measure the mass of an electron, since R, B, and V_a are measurable and q is known.

In this investigation you will vary the current in the solenoid (thus varying the field strength B) and measure the radius of curvature R of the path of the electrons. You will then average your results to find the mass of an electron.

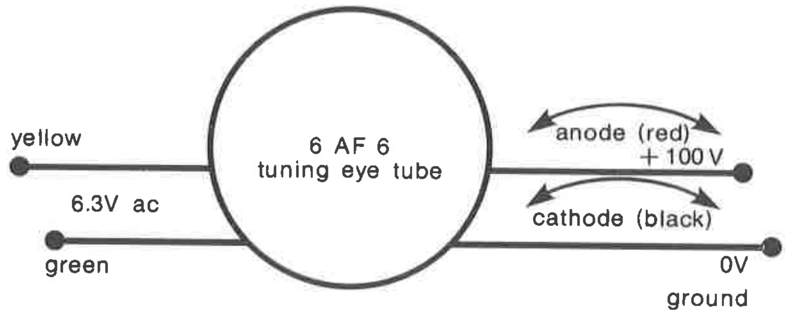
Apparatus

6AF6 tuning eye tube
50 W rheostat
solenoid (use the one you used in Investigation 11)
power supply: 100 to 250 V dc, 6 V dc, and 6.3 V ac
assorted cylinders not made of iron, steel, nickel, etc. (such as wooden dowels) with diameters between 0.5 cm and 2 cm
ammeter
voltmeter

Procedure

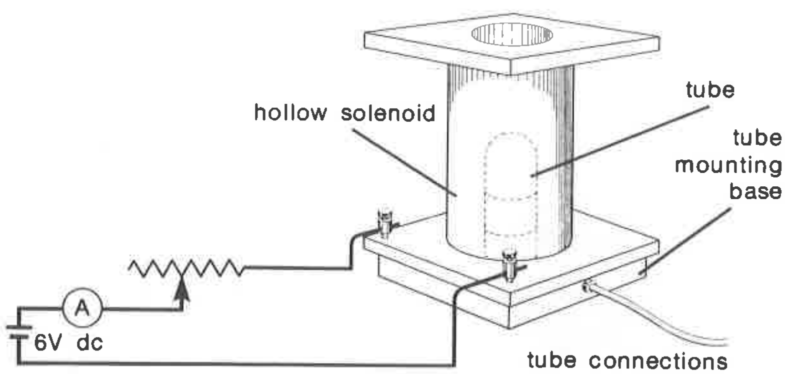
1. Connect your tuning eye tube to the power supply (see figure #054).

#054 Tuning eye tube connected to power supply



2. Connect the solenoid through a rheostat to the power supply. Set the rheostat for minimum current.
3. Place the hollow solenoid over the tube (see figure #055). Turn on the power supply and observe the change in the beam pattern of the tube as the current in the solenoid is slowly increased.

#055 Solenoid and rheostat connected to power supply



4. Choose the largest of your cylindrical objects and hold it inside the solenoid and over the top of the tube so that its curvature can be directly compared to the curvature of the electron beam in the tube. Adjust the current through the solenoid until these curvatures match as closely as possible. Record the current I and the radius of curvature R.

5. Repeat procedure 4 using the other cylindrical objects.
6. Repeat the experiment using potential differences of 150 V and then 250 V between the cathode and the anode.

Analysis of Data

1. For each set of measurements calculate
   (a) the magnetic field strength B from the current I, using the calibration constant k you measured in Experiment 11 or the theoretical expression $B = \mu\left(\frac{N}{l}\right)I$.
   (b) the electron's mass m.
2. From your results, find the average value for the mass of the electron.

Interpretation of Data

1. What is the maximum percentage difference between your values of the mass of an electron and the average value? (See Appendix 5.)
2. Compare your average value of the mass of the electron to the accepted value of $m = 9.1 \times 10^{-31}$ kg. Calculate the percentage difference (see Appendix 5).
3. List as many as you can of the assumptions made in calculating the mass of the electron in this investigation. For example, one assumption is that all the electric potential energy lost by the electrons in travelling to the anode is converted to kinetic energy.

Additional Activities

1. The principle of operation of the tuning eye tube is also used in an instrument called a mass spectrograph. Describe this instrument and show how it is used to determine the atomic masses of different isotopes of an element and their relative abundance.
2. Describe the working principles of a cyclotron, a device used to accelerate charged particles to velocities close to the speed of light.

Practice Problems

1. (a) A magnetic field of magnitude 3.4×10^{-3} T deflects electrons into a circular path of radius 1.4 cm. The electrons have been accelerated through a potential difference of 250 V. If the charge of an electron is 1.6×10^{-19} C, what is the mass of an electron?

(b) If protons were accelerated by the same potential as the electrons in part (a) and deflected by the same field, what would be the radius of their orbit?

(c) If the electrons were found to revolve clockwise in their circular orbits, explain why the protons would revolve counter-clockwise.

2. In a proton storage ring, very fast protons are kept in a circular orbit by a uniform magnetic field applied at right angles to the plane of the orbit.

   (a) Show that the proton's period of revolution T is $\frac{2\pi m}{qB}$ where m and q are the mass and charge of a proton and B is the magnetic field strength. (Use the fact that the magnetic force is a centripetal force and remember that the speed around the circle is constant.)

   (b) Calculate the period of revolution if the magnetic field strength is 1.2×10^{-2} T.

   (c) If the protons have been accelerated through a potential difference of 3.0×10^{6} V, what strength of magnetic field is needed to keep them in an orbit of diameter 8.0 cm?

   (d) How does your answer to (c) compare with the value of the Earth's magnetic field near the surface (roughly 3.0×10^{-5} T)?

3. An instrument called a velocity selector has charged particles passing through a region where there is a uniform electric field $\vec{E}$ perpendicular to a uniform magnetic field $\vec{B}$. The particles travel at right angles to both $\vec{B}$ and $\vec{E}$ so that the electric force and the magnetic force on them are in opposite directions. If B is 0.35 T and E is 6.0×10^{4} V/m, at what speed would particles pass through undeflected?

True science teaches, above all, to doubt, and to be ignorant.

Miquel de Unamuno

Investigation 13:

Electromagnetic Induction

In previous investigations you have learned that moving charges (currents) have magnetic fields around them and that charges moving in a magnetic field experience a deflecting force. In this investigation you will observe that a changing magnetic field can cause charges to move in a conductor, or, in other words, that a current can be induced in a conductor. If there is an induced current there must be an induced emf. This phenomenon is called **electromagnetic induction**. It is the basis of operation of electric generators and transformers and it also occurs in motors.

Part A

This part of the investigation focuses on the factors that affect the induction of current in a solenoid.

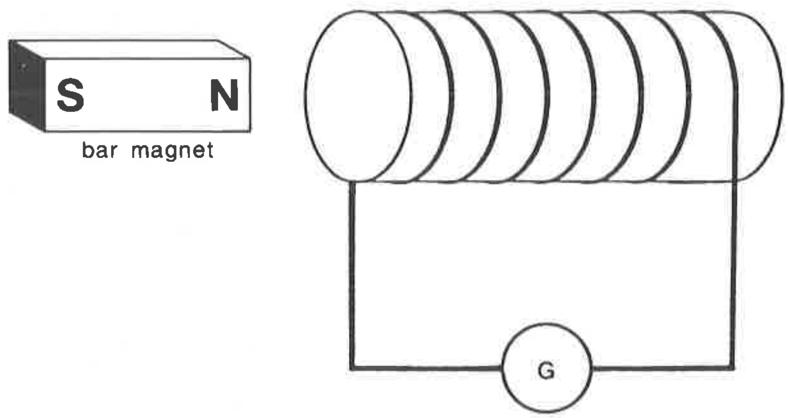
Apparatus

solenoid
bar magnets
galvanometer
plotting compass
1.5 V dry cell
50 W rheostat
20 cm length of resistance wire

Procedure

1. Connect the galvanometer to the solenoid (see figure #056).

 # 056 Galvanometer connected to a solenoid



2. Hold the N end of the magnet near one end of the solenoid; then push it quickly into the coil. Record the direction in which the galvanometer needle swings. Quickly withdraw the magnet and again record the direction of swing of the needle.
3. Repeat step 2 of the procedure using the S end of the magnet.
4. Examine the dependence of the magnitude of the induced current on
   (a) the speed of insertion of the magnet,
   (b) the number of coils of the solenoid which surround the magnet,
   (c) the strength of the magnet inserted,
   (d) the orientation of the magnet with respect to the solenoid (e.g., perpendicular to the plane of the coils, parallel to the plane of the coils, inside the solenoid, outside the solenoid).

Discussion of Results

1. Describe the ways in which you could increase the magnitude of the induced current in a solenoid.
2. Describe the ways in which you could change the direction of the induced current in a solenoid.

3. The current induced in a solenoid produces a magnetic field. How does the induced field affect the N pole of the magnet entering the solenoid? Does the direction of the induced field change when the magnet is withdrawn from the solenoid? How does the field affect the N pole of the magnet as it is withdrawn? Does the same apply to the S pole as it enters and leaves the solenoid? State a general rule which you can use to give the direction of the induced current.

Part B

A motor consists of a coil of wire (the armature) in a magnetic field. When a power source is connected to the motor, there is a current through the armature and the armature experiences a magnetic force. As a result, it spins. However, once the armature is spinning in the magnetic field, a current is induced in it (just as it was in Part A; it does not matter whether the magnet moves or the coil).

This part of the investigation focuses on the changes that occur in the current through a motor when its armature spins.

Apparatus

galvanometer
voltmeter
ammeter
small dc motor
1.5 m length of thread
several washers each having the same mass
stopwatch

Procedure

1. Make sure that the shaft of your motor has a fairly large diameter. If necessary, drill a piece of dowelling and glue it to the existing shaft to make it larger.

2. Connect the terminals of the motor to a galvanometer. Spin the shaft of the motor by hand and observe the deflection of the galvanometer needle. Reverse the direction of spin and note the deflection again. Spin the shaft faster and notice any change in the induced current.

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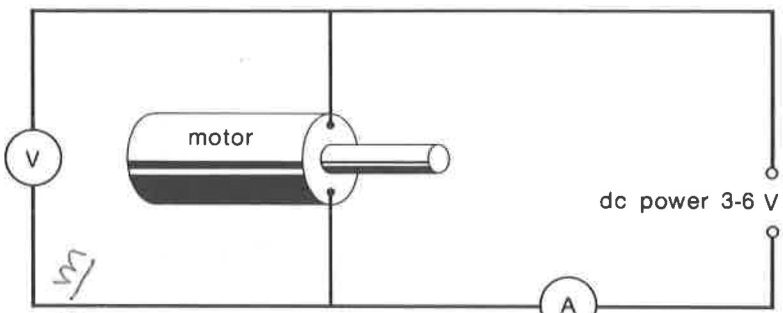
3. Connect the motor, a voltmeter, and an ammeter to a power supply (see figure #057). Adjust the voltage so that the motor spins at nearly maximum speed.

#057 Motor, voltmeter, and ammeter connected to a power supply



4. Hold the shaft of the motor so that it cannot turn and measure the voltage across the motor and the current through it.
5. Attach one end of a 1.5-m length of thread to the shaft of the motor in such a way that the thread will not slip while the shaft is rotating. Place the motor so that the shaft overhangs the edge of a table. Reconnect the motor to the power supply, the ammeter, and the voltmeter.
6. Attach a washer to the free end of the thread. Turn on the motor and record the time required for it to lift the washer 1.0 m. Also record the voltage across the motor and the current through it.
7. Repeat step 6 of this procedure using 2, 3, 4, and then 5 washers of the same mass.

Data Analysis

1. Calculate the resistance of the motor's coil from the voltage across it and the current through it when the shaft is prevented from rotating.
2. Make a table showing the speed v at which the washers rise and the corresponding back emf V_b of the motor. ($V_b = V_a - IR$ where V_a is the voltage applied to the motor, I is the current through the motor, and R is the resistance of the motor.)
3. Plot V_b versus v (see Appendix 3) and determine the relationship between them (see Appendix 4).

Discussion of Results

1. How does the direction of the induced current in the armature compare with the direction of the current from the power source? Explain how you know.

2. How does the speed of the washers depend on the frequency of rotation of the motor?

3. How does the back emf depend on the frequency of rotation of the motor? (See Appendix 7.)

4. Using the diameter of the motor's shaft, calculate the exact relationship between the speed v of the washers and the frequency f of rotation of the motor. At what speed v (according to your graph) would the current through the motor be zero? At what frequency of rotation would this happen? If you spun the motor faster, what would be the direction of the current through the motor? Why? Would you still call it a motor? Explain.

Additional Activities

1. Attach a thin strip of aluminum (or other non-magnetic metal) to an air track cart so that the strip is vertical. Arrange a strong horseshoe magnet over the track so that the strip can pass between the poles of the magnet without contact. Send the cart slowly towards the magnet. Observe the effects and explain them in terms of magnetic induction. Also examine the construction of the damper on a triple-beam balance and explain its operation. Find out how a house watt-hour meter records the use of electrical energy or how a car speedometer operates.

2. Examine a demonstration transformer. Find out the mathematical formula that applies to its operation.

3. Determine the factors that affect the speed of operation of a St. Louis motor.

4. Find out the principles of operation of an induction coil (such as the power supply for a gas discharge tube or Crookes tube).

5. Examine a linear induction motor and explain its principles of operation.

Practice Problems

1. When a motor with a stationary coil is connected to a 4.5 V power supply, there is a current of 3.0 A through it. When the coil is released, the current drops to 1.8 A.

   (a) What is the resistance of the coil?

   (b) What is the back emf in the motor at this rate of rotation?

   (c) The load on the motor is reduced, allowing the speed of the motor to double. What is the new current through the motor?

$$\mathcal{E} = \frac{d\Phi_B}{dt}$$

$$\mathcal{E} = \frac{dB \cdot A}{dt}$$

$$\frac{R}{2\pi r}$$

$$2\pi r f = \frac{R}{t}$$

2. The efficiency of a motor is measured as the ratio of output power to input power. The input power is IV_a, where V_a is the applied voltage and I is the current through the motor, and the power lost as heat in the armature is I^2R, where R is the resistance of the motor.

   (a) Show that the output power can be written as IV_b, where V_b is the back emf, and that the efficiency of the motor is $\frac{V_b}{V_a}$.

   (b) Show that the efficiency of a motor is directly proportional to the frequency of rotation f and decreases linearly as the current I through the motor increases.

   (c) When a small dc motor with a resistance of a 1.2 Ω is connected to a 9.0 V power supply, the current through it is 4.2 A. What is its efficiency?

3. Faraday's law of electromagnetic induction states that the magnitude of the emf induced in a coil is proportional to the rate of change of magnetic flux Φ through the coil and to the number of turns N in the coil. The average emf induced during a time interval of $\triangle t$ is $E = -N\frac{\triangle\phi}{\triangle t}$. If the magnetic field B is uniform then $\phi = BA$, where A is the area of the coil. A circular coil of wire containing 30 turns of radius 12 cm is rotating at 15 r/s in a uniform field of strength 0.30 T.

   (a) What is the maximum area of the coil that could be perpendicular to the field?

   (b) Assume that the minimum area perpendicular to the field is zero. How many times per revolution does the area perpendicular to the field change between maximum and minimum?

   (c) Calculate the change in the magnetic flux through the coil for one quarter of a revolution.

   (d) Calculate the average emf induced in the coil during one quarter of a revolution.

There is something fascinating about science. One gets such wholesale returns of conjecture out of such trifling investment of fact.

Mark Twain

AC Circuits and Electronics

Investigation 14:

Capacitance

A capacitor is a device for storing electric charge. As charge is added to a capacitor, the potential difference across the capacitor goes up in direct proportion. At any instant, the charge Q on the capacitor can be given by $Q = CV$, where V is the potential difference across the capacitor and C is a constant called the **capacitance** of the capacitor.

When two capacitors of capacitance C_1 and C_2 are connected in series, their equivalent capacitance C can be shown to be given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

When they are connected in parallel, their equivalent capacitance C is

$$C = C_1 + C_2.$$

One use for capacitors is in relaxation oscillator circuits. You will study such a device containing a neon tube. A neon tube has the characteristic of not letting current through it until the potential difference across it reaches a certain **striking voltage.** When placed in parallel with a capacitor and a power source, the neon tube will flash with a frequency which is characteristic of the circuit. This type of circuit is used in most strobe lights and "blinky" lights.

This investigation is in two parts. In the first part you will study the discharge of a capacitor. In the second part you will study a circuit containing a neon tube.

Part A:

In this section of the investigation you will place a charge on a capacitor in order to study its discharge.

Apparatus

5000 μF capacitor
10 kΩ resistor
20 kΩ/V voltmeter or better (even with 20 kΩ/V, there is an 8% error in the time constant)

or

5.0 μF capacitor
10 MΩ resistor
vacuum-tube voltmeter or a cathode ray tube

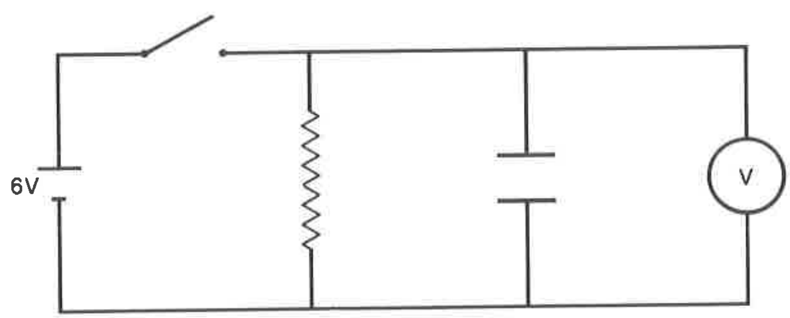
6 V power supply
stopwatch
switch

CAUTION: It is possible to receive a severe electric shock from an undischarged capacitor.

Procedure

1. Connect the power supply, the cut-off switch and the resistor in series. Then connect the capacitor and the voltmeter in parallel with each other and with the resistor (see figure #058). Be sure to observe the correct polarities. If you use a cathode ray tube as a voltmeter, set it up as you did in Investigation 8 and use the *X*-plates as the voltmeter terminals. You can then record the deflection of the electron beam instead of voltage.

#058 Circuit for step 1



2. Close the switch and record the maximum voltage V_o reached by the voltmeter. When the switch is opened the capacitor will discharge (as shown by the dropping voltage).
3. Select a convenient voltage that is lower than V_o. Start the stopwatch as you open the switch and stop it when this voltage is reached. Record the time.

4. Recharge the capacitor and repeat step 3 for a second, lower voltage.
5. Repeat this process at least ten times.

Data Analysis

1. Graph V versus t (see Appendix 3) and determine the relationship between them. (See Appendix 4).
2. From your equation, calculate the time taken for the voltage across the capacitor to drop to 36.7879...% ($1/e$) of its original value. This time is called the **time constant** of the circuit.
3. From your equation, calculate the time taken for the voltage to drop to 13.5335... % ($1/e^2$) of its original value and then the time for it to drop to 1.8316... % ($1/e^3$) of its original value. How do these times compare with the time constant of the circuit?
4. Calculate the product RC, where R is the resistance of the resistor through which the capacitor discharged and C is the capacitance of the capacitor. What is its unit? How does its value compare with the time constant of the circuit?

Part B

In this section of the investigation you will set up a neon tube oscillator circuit to study the factors that influence the time constant of a circuit containing a capacitor.

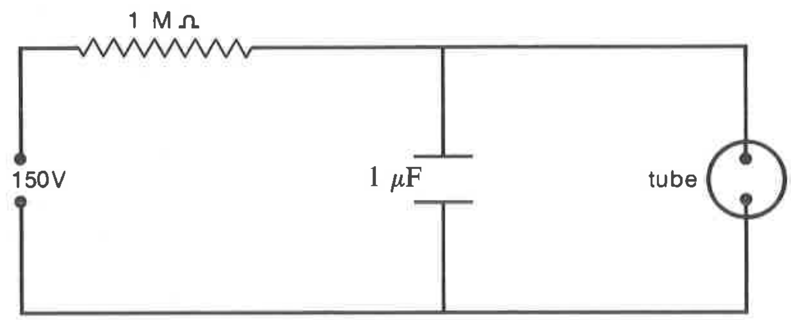
Apparatus

0.5 μF, 1.0 μF, 2.0 μF capacitors
1.0 MΩ, 2.0 MΩ, 5.0 MΩ resistors
150 V power supply
neon tube
stopwatch
voltmeter

Procedure

1. Set up the oscillator circuit as shown in figure #059. Record the period of the flashes of the neon tube.

#059 Oscillator circuit for step 1



2. Vary the resistor and the capacitor to obtain nine different combinations. Measure the period of the flashes in each case.

3. Using single resistors, combine capacitors in both series and parallel connections. Determine the period of the flashes in each case.

4. Investigate the effect of lower power-source voltages on the operation of the neon tube.

Data Analysis

1. Graph the period T against the product RC of the resistance and the capacitance (see Appendix 3).

2. Determine the relationship between the period of the neon tube and the time constant RC of your circuit. (See Appendix 4.) How does the period of the flashes depend on the time constant of the circuit? (See Appendix 7.) What, theoretically, might the slope of your graph depend on?

3. From the equation of your graph, what is the equivalent capacitance of each combination of capacitors you used? What is the theoretical capacitance? What is the percentage difference between them?

Additional Activities

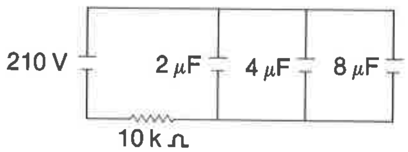
1. If a "blinky" light is available, check its circuit diagram and specifications. Does the circuit resemble your own relaxation oscillator circuit? Measure the flash rate of the "blinky"; then calculate the flash rate from the specifications.

2. Obtain a "variable condenser" from a radio parts dealer. Explain the method used to control the capacitance of the condenser. Determine the minimum and maximum capacitance of the condenser with a suitable RC circuit containing a neon tube. Check the appropriate theory with your results.

Practice Problems

1. The time constant of a circuit containing a capacitor and a resistor is 8.4 s. If the resistance of the resistor is 5.0 MΩ, what is the capacitance of the capacitor?

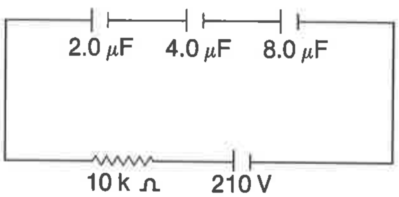
#060 Diagram for Problem 2



2. (a) Calculate the equivalent capacitance of the three capacitors shown in diagram #060.

   (b) For figure #060, calculate the voltage across each of the capacitors when they are fully charged.

#061 Diagram for Problem 3



3. (a) Calculate the equivalent capacitance of the three capacitors shown in figure #061.

   (b) For figure #061, calculate the voltage across each of the capacitors when they are fully charged.

4. A circuit contains a capacitor of capacitance 2.5 μF and a resistor of resistance 1.5 MΩ in series with a power source of voltage 200 V. When the switch is closed, how long does it take the capacitor to reach a potential of 180 V?

Insofar as the propositions of mathematics refer to reality they are not certain, and insofar as they are certain they do not refer to reality.

Albert Einstein

Appendix 14

Answers to Practice Problems

Investigation 1

1. 4.0 s
2. (a) 4.0 s
   (b) 1.6×10^2 m
3. (a) 51 m/s
   (b) 61 m/s
   (c) 10 s
   (d) 1.3×10^2 m
   (e) 6.4×10^2 m
4. (a) 39 m/s
   (b) 1.3×10^2m

Investigation 2

1. (b) 5.1×10^4 kg•m/s at 62° South of West
   (c) 10 m/s at 62° South of West
2. (a) 6.5 m/s at 38° West of North
   (d) $v_1 = 10$ m/s, $v_2 = 6.0$ m/s
3. 6.7 m/s at 27° East of North
4. (a) 1.3 m/s
   (b) first 0.43 m/s in the opposite direction, second 2.6 m/s in the original direction
5. (a) 2.6 m/s
   (b) due North
   (c) 50%

6. (a) 4.7 m/s
   (b) 31° to the original direction of the ball
   (c) 6.4%
7. 31 km/h due Northeast
8. 5.9 m/s 31° South of West
9. 4.9 m/s 27° North of West, no
10. (a) 4.9 m/s 19° North of West
    (b) Yes
11. (a) 0.80 m/s 54° North of West
    (b) 2.88 J; 2.87 J; yes
12. first 1.2 m/s, second 1.5 m/s

Investigation 3

1. The forces are not in equilibrium.
2. $\Sigma\vec{F}_x = 0.3$ N right, $\Sigma\vec{F}_y = 0.4$ N up
3. (a) 12.3 N at a 128° angle to the 5.0 N force
   (b) 12.3 N at 6.8° to the left of the positive y axis
   (c) $F_x = 1.5$ N, $F_y = -12.2$ N
   (d) 12.3 N at 6.8° to the right of the negative y axis
4. horizontal 29 N, other 58 N
5. 2.29×10^3 N

Investigation 4

1. (a) $F = 18$ N
   (b) $F = 9.0$ N
2. $x = 1.0$ m
3. (a) $F = 30$ N
   (b) $x = 32$ cm
4. (a) $F = 178$ N
   (b) $\vec{R} = 182$ N 12.7° above the horizontal

Investigation 5

1. (a) 8.4 m/s
   (b) 35 m/s^2
   (c) 18 N
   (d) 1.9 kg
   (e) 74°
   (f) $L = 2.1$ m
2. (a) 0.52 m/s
   (b) 2.3 times as fast
3. (a) 5.5×10^3 s
   (b) 7.7×10^3
4. (a) 2.0×10^{-4} Hz = 0.72 r/h = 17 r/d

Investigation 6

1. 2.28×10^{10} s
2. 2.7×10^{12} m
5. $\theta = 34°$ (for Vancouver)

Investigation 7

1. (a) ¼
   (b) 9
   (c) 9/4
2. 4.0×10^{-6} C
3. (a) 2.6×10^{-7} N
   (b) 1.1×10^{-46} N
   (c) 2.2×10^{39}
4. (a) 2.0×10^{-7} N to the left
   (b) 2.0×10^{-7} towards the top of the page

Investigation 8

1. 1.3×10^7 m/s
2. (a) 8.0×10^{-17} J
   (b) 1.3×10^7 m/s
   (c) 7.5×10^3 V/m
   (d) 1.2×10^{-15} N
   (e) 1.3×10^{15} m/s^2
   (f) 4.5×10^{-9} s
   (g) 1.4×10^{-2} m
   (h) 6.0×10^6 m/s
   (i) 1.5×10^{-8} s
   (j) 0.10 m
3. (a) 8.0×10^{-17} J
   (b) 3.1×10^5 m/s
   (c) 7.5×10^3 V/m
   (d) 1.2×10^{-15} N
   (e) 7.1×10^{11} m/s^2
   (f) 2.0×10^{-7} s
   (g) 1.4×10^{-2} m
   (h) 1.4×10^5 m/s
   (i) 6.5×10^{-7} s
   (j) 0.10 m

Investigation 9

1. 15 Ω
2. $R = 2.4$ Ω
3. 0.25 A
4. 0.99 Ω
5. (a) 1.4 V
   (b) 0.1 Ω
   (c) 1.4 V

Investigation 10

1. (a) 7.2×10^{-15} N
   (b) 7.9×10^{15} m/s^2
   (c) 2.1×10^{12} m/s^2
2. 3.0 cm
3. 2.3×10^2 V
4. (a) 1.2×10^{-30} kg

Investigation 11

1. (a) 0.19 T
   (b) No force
   (c) 1.7×10^{-2} N
2. 0.47 A
3. 2.6 A

Investigation 12

1. (a) 7.3×10^{-31} kg
   (b) 68 cm
2. (b) 5.6×10^{-6} s
   (c) 6.3 T
   (d) 2.1×10^5 times as strong
3. 1.7×10^5 m/s

Investigation 13

1. (a) $1.5\ \Omega$
   (b) 1.8 V
   (c) 0.6 A
2. (c) 44%
3. (a) $4.5 \times 10^{-2}\ m^2$
   (c) 1.4×10^{-2} Wb
   (d) 24 V

Investigation 14

1. $1.7\ \mu F$
2. (a) $14\ \mu F$
   (b) 210 V
3. (a) $1.1\ \mu F$
   (b) 120 V, 60 V, 30 V
4. 8.6 s

Investigation 15

1. −13.6 eV, −3.40 eV, −1.51 eV, −0.850 eV, −0.544 eV
4. 1.89 eV, 659 nm; 2.55 eV, 488 nm; 2.86 eV, 436 nm; 3.02 eV, 412 nm

Investigation 16

1. $2.4 \times 10^3\ kg/m^3$
2. 89%
3. 4.7×10^5 N, 4.4×10^5 N
4. 0.150 kg